EECS 120 **Fall 2004**

Demo 8 Solutions

(1) Problem 7.49 of OWN:

(a) Let $x_1[n]$ and $x_2[n]$ be two inputs with corresponding outputs $y_1[n]$ and $y_2[n]$ respectively. Now, suppose the new input to this system is $\alpha x_1[n] + \beta x_2[n]$. Then, the corresponding output is given by, $y[n] = \alpha x_1[\frac{n}{N}] + \beta x_2[\frac{n}{N}]$ if $n = 0\ , \pm N, \pm 2N, \ldots$ and 0 otherwise. From this it is clear that $y[n] = \alpha y_1[n] + \beta y_2[n]$ and hence the system is **linear**.

(b) The system A is not **time invariant**. We can see this from the following example. Let $x[n] = 1$, for $n = 0, 1$ and $x[n] = 0$ otherwise. Also assume that N=2. Thus, the output corresponding to $x[n]$ will be $y[n] = 1$ if $n = 0, 2$ and $y[n] = 0$ otherwise. Now, let $x_1[n] = x[n-1] = 1$ if $n = 1, 2$ and 0 otherwise. The corresponding output is $y_1[n] = 1$ if $n = 2, 4$ and $y_1[n] = 0$ otherwise. Thus, $y_1[n] = y[n-2] \neq y[n-1]$ and hence this shows that the system is not time invariant.

(c)Refer to Fig. 1 and Fig. 2

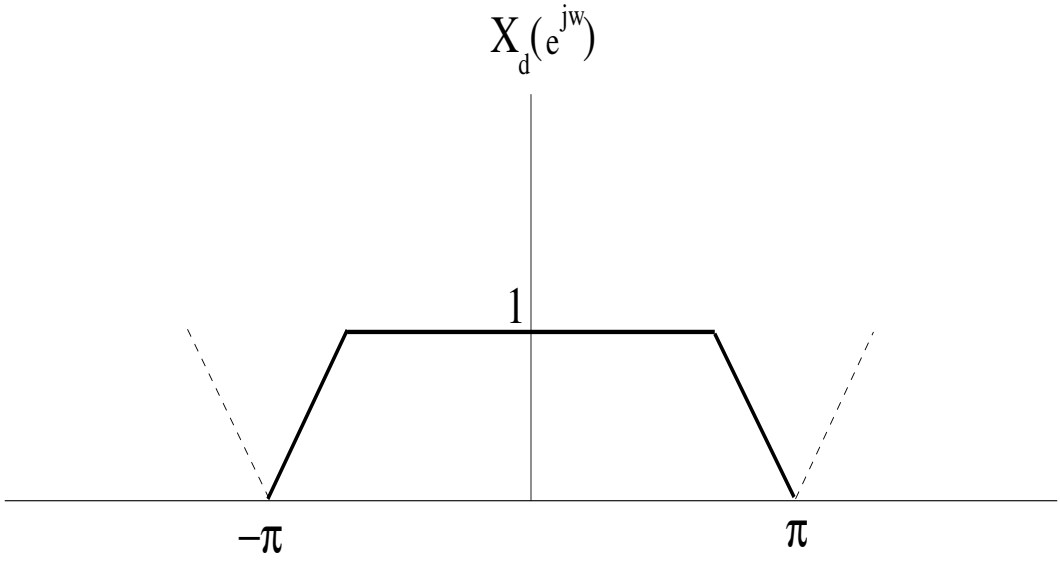

Figure 1: Spectrum given in the problem

(d) Refer to Fig. 3.

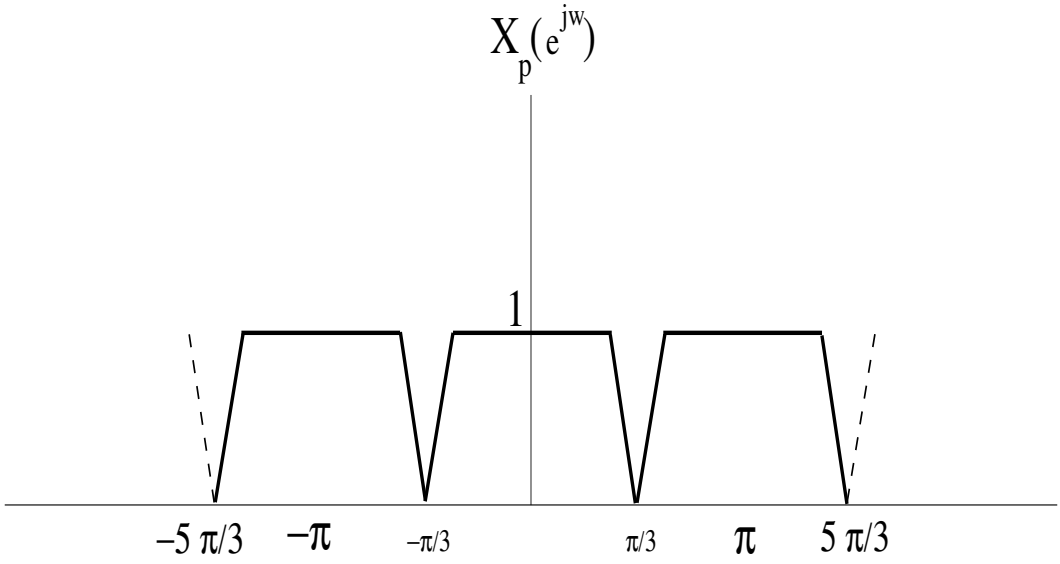

Figure 2: Spectrum of the interpolated signal

(2) Problem 8.8 of OWN:

(a) From the system diagram we can write the following input output relationship,

$$y(t) = [x(t) * h(t)] \cos \omega_c t + x(t) \sin \omega_c t$$

Observe that $H(j\omega)$ is conjugate symmetric, and hence $h(t)$ must be real. Therefore, from this observation and from the system equation written above it is clear that $y(t)$ has to be real when $x(t)$ is real.

(b) Since $x(t)$ is real its Fourier transform will be conjugate symmetric, i.e., $X(-j\omega) = X^*(j\omega)$. Hence we can write $X(j\omega)$ as

$$X(j\omega) = X_+(j\omega) + X_+^*(-j\omega)$$

where $X_+(j\omega)$ is the positive half of the Fourier transform of $X(j\omega)$. Therefore, $X(j\omega)H(j\omega) = -jX_+(j\omega) + jX_+^*(-j\omega)$. Now, writing the system equation in the frequency domain,

$$\begin{aligned}
Y(j\omega) &= \frac{1}{2\pi}\left[(X(j\omega)H(j\omega)) * (\frac{2\pi}{2}[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)]) + X(j\omega) * \frac{2\pi}{2j}[\delta(\omega-\omega_c)-\delta(\omega+\omega_c)]\right] \\
&= \frac{1}{2}\left[-jX_+(j(\omega-\omega_c)) + jX_+^*(-j(\omega-\omega_c)) + -jX_+(j(\omega+\omega_c)) + jX_+^*(-j(\omega+\omega_c))\right] \\
&\quad -\frac{1}{2}\left[jX_+(j(\omega-\omega_c)) + jX_+^*(-j(\omega-\omega_c)) - jX_+(j(\omega+\omega_c)) - jX_+^*(-j(\omega+\omega_c))\right] \\
&= j[X_+^*(-j(\omega+\omega_c)) - X_+(j(\omega-\omega_c))]
\end{aligned}$$

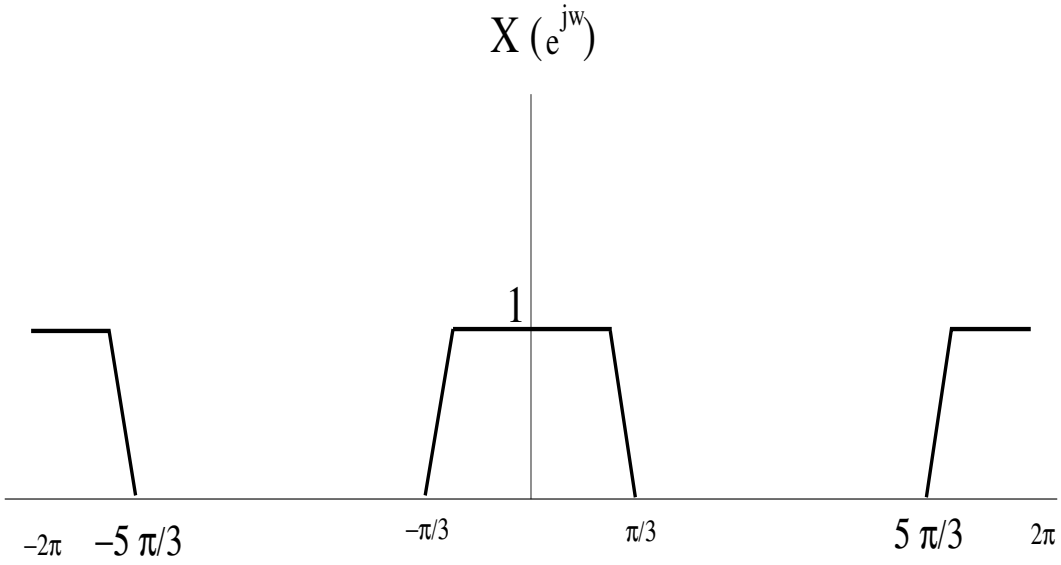

Figure 3: Spectrum of the upsampled signal

From the last equation it is clear that we can reconstruct $x(t)$ from $y(t)$. This can be done in the following way. First pass $y(t)$ through a filter with frequency response $-H(j\omega)$, and then multiply the output by $\cos \omega_c t$. Finally, pass the output of the multiplier through a low pass filter with cut off frequency ω_M and gain 2. To see how this works, first let the output of the multiplier be denoted by $z(t)$, then

$$\begin{aligned} Z(j\omega) &= \frac{1}{2\pi}[Y(j\omega)(-H(j\omega))] * \left[\frac{2\pi(\delta(\omega-\omega_c)+\delta(\omega+\omega_c))}{2}\right] \\ &= [X_+(j(\omega-\omega_c)) + X_+^*(-j(\omega+\omega_c))] * \left[\frac{\delta(\omega-\omega_c)+\delta(\omega+\omega_c)}{2}\right] \\ &= \frac{X_+^*(-j\omega)+X_+(j\omega)}{2} + \text{higher frequency terms} \\ &= \frac{1}{2}X(j\omega) + \text{higher frequency terms} \end{aligned}$$

Therefore, the output of $z(t)$ through the low pass filter will give us back $x(t)$.

(3) Problem 8.12 of OWN:
We assume that the Fourier transform of each signal $x_i(t)$ satisfies, $X_i(j\omega) = 0$ for $|\omega| \geq 2000\pi$. In order to recover the signal $x_i(t)$ from the modulated signal we have to impose some constraint on the frequency of the carrier $c(t)$. Since the carrier $c(t)$ is periodic with period T, its fourier transform

$C(j\omega)$ is $2\pi \sum_k a_k \delta(\omega - k\omega_c)$, where a_k are the Fourier series coefficients of $c(t)$, i.e., $a_k = \frac{\sin(k\omega_c \Delta/2)}{\pi k}$ and $\omega_c = \frac{2\pi}{T}$. Therefore for perfect reconstruction we must require that the bandwidth of the signal must be less than ω_c, i.e., $\omega_c > 4000\pi$, see Figure 8.24 of the text. This implies that $T < \frac{1}{2}10^{-3}$. Also, if we have to time division multiplex 10 signals with a carrier signal of period T, the width of the pulse $\Delta \leq \frac{T}{10}$. Combining with the inequalities we have

$$\Delta \leq \frac{T}{10} < \frac{\frac{1}{2}10^{-3}}{10} = 0.5 \cdot 10^{-4}$$

Hence the largest possible value of $\Delta = 0.5 \cdot 10^{-4}$.

(4) Balanced modulator:

(a)Refer to Figs. 4, 5 and 6. The figures are drawn assuming that $\omega_c > 3\omega_M$.

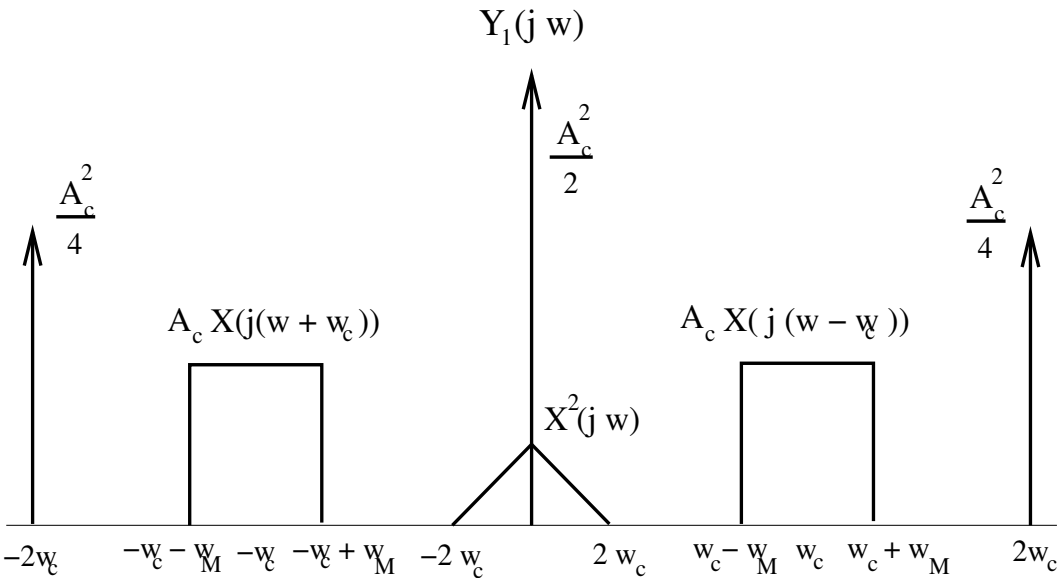

Figure 4: Spectrum of $Y_1(j\omega)$

(b) The input-output relation ship for the above system is given by

$$\begin{aligned} y(t) &= y_1(t) - y_2(t) \\ &= [A_c \cos(\omega_c t) + x(t)]^2 - [A_c \cos(\omega_c t) - x(t)]^2 \\ &= 4x(t) A_c \cos(\omega_c t) \end{aligned}$$

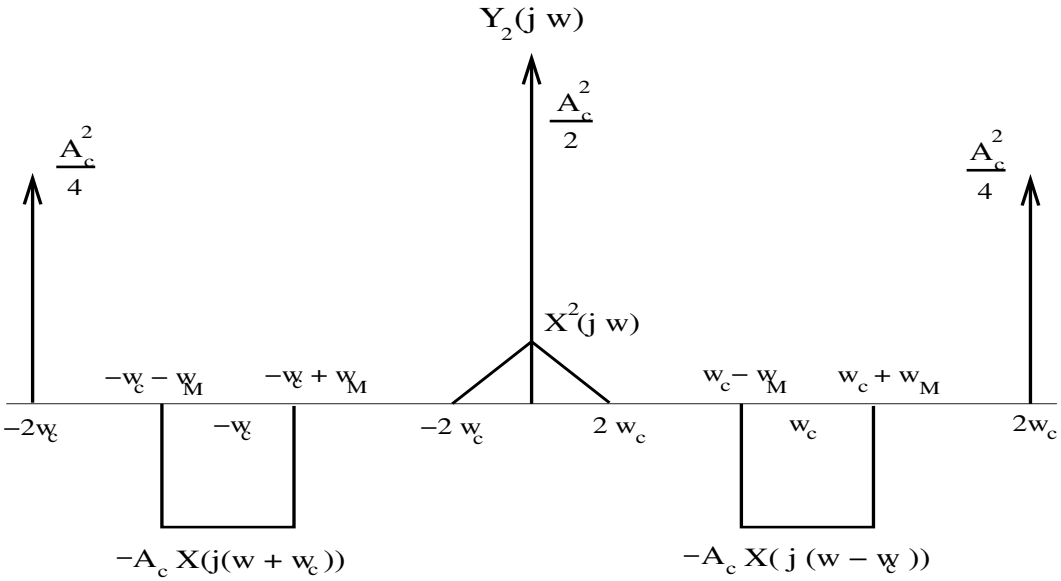

Figure 5: Spectrum of $Y_2(j\omega)$

Hence the output $y(t)$ gives DSB-SC AM of the modulating signal onto the carrier.

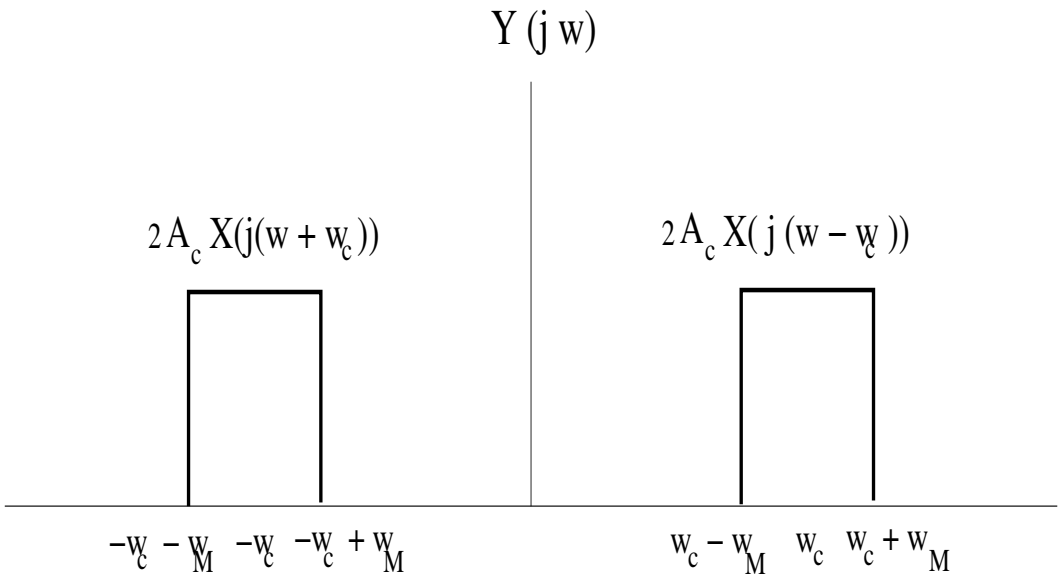

Figure 6: Spectrum of $Y(j\omega)$