EECS 120 **Fall 2004**

Sample Midterm 2

• The exam is for one hour and 50 minutes.

• The maximum score is 100 points. The maximum score for each part of each problem is indicated.

• The exam is closed-book and closed-notes. Calculators, computing, and communication devices are NOT permitted.

• Four double-sided sheets of notes are allowed. These should be legible to normal eyesight, i.e. the lettering should not be excessively small.

• No form of collaboration between students is allowed.

1. *(10 points)* State whether the following are true or false. In each case, give a brief explanation. A correct answer with a correct explanation gets 5 points. A correct answer without a correct explanation gets 2 points. A wrong answer gets 0 points.

   (a) Frequency modulation satisfies the superposition principle, i.e. for $i = 1, 2$ if $u_i(t)$ denotes the signal resulting from frequency modulating $m_i(t)$ onto the carrier $A_c \cos(\omega_c t)$ then $au_1(t)+bu_2(t)$ is the signal resulting from frequency modulation $am_1(t)+bm_2(t)$ onto the carrier.

   (b) If a signal $x(t)$ has Fourier transform $X(j\omega)$ such that

   $$X(j\omega) = 0 \text{ if } \mid \omega \mid \leq 2\omega_M$$

   and

   $$X(j\omega) = 0 \text{ if } \mid \omega \mid \geq 3\omega_M$$

   then $x(t)$ can be recovered from the sequence of its samples at the times $nT, n \in \mathbf{Z}$, where $\frac{2\pi}{T} = 3\omega_M$.

2. *(10 points)*

   Let $\omega_0 << \omega_c$. The modulating signal

   $$m(t) = \cos(\omega_0 t) - 0.7 \sin(2\omega_0 t)$$

   is USSB amplitude modulated onto the carrier $A_c \cos(\omega_c t)$. Find an explicit expression for the modulated signal $u(t)$.

   Hint : First draw a picture of the Fourier transform of $u(t)$.

3. *(10 points)*

A lowpass signal $m(t)$ which is bandlimited to $(-\omega_M, \omega_M)$ is modulated onto the carrier $A_c \cos(\omega_c t)$ using DSB-SC amplitude modulation. Here $\omega_M << \omega_c$. The resulting signal is modulated onto the carrier $A_d \cos(\omega_d t)$, where $\omega_c << \omega_d$, using USSB amplitude modulation. Let $x(t)$ denote the resulting signal.

Assume that the Fourier transform of $m(t)$ is

$$M(j\omega) = \begin{cases} e^{j\omega} & \text{if } |\, \omega \,| < \omega_M \\ 0 & \text{otherwise} . \end{cases}$$

Sketch the Fourier transform $X(j\omega)$. *Note* : Sketch both the magnitude and the phase of the Fourier transform.

4. *(5 + 10 + 3 + 10 + 2 points)*

   For a linear time invariant system, it is known that the system function is given by

   $$H(s) = \frac{6(s-4)}{(s+3)(s^2-4s+13)} .$$

   (a) Draw the pole-zero diagram for $H(s)$.

   (b) Suppose that you are told that the system is causal. Find the impulse response $h(t)$ of the LTI system.

   (c) Suppose you are told that the system is causal. Is it stable ? Explain your answer.

   (d) Suppose that you are told that the system is stable. Find the impulse response $h(t)$ of the LTI system.

   (e) Suppose you are told that the system is stable. Is it causal ? Explain your answer.

5. *(10 points)*

   Let $x(t)$ and $y(t)$ be signals, each of which is bandlimited to $(-\omega_M, \omega_M)$. Let $a > 1$ be a real number and let

   $$z(t) = \cos(a\ \omega_M t) + y(t) .$$

   Let $v(t) = x(t)z(t)$. The signal $v(t)$ is periodically sampled with period T.

   For what values of a and T is it possible to recover *both* $x(t)$ and $y(t)$ from the samples $v(nT), n \in \mathbf{Z}$?

6. *(10 points)*

   Let $x[n]$ be a discrete time signal, with discrete time Fourier transform $X(e^{j\omega})$. Recall that

   $$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} , \quad \text{and}$$

   $$x[n] = \frac{1}{2\pi} \int_{<2\pi>} X(e^{j\omega})e^{j\omega n} d\omega .$$

   Assume that

   $$X(e^{j\omega}) = 0 , \quad \text{if } \omega_M < |\, \omega \,| \leq \pi .$$

Let $H(e^{j\omega})$ denote the ideal low pass filter

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \Pi(\frac{\omega - 2\pi n}{2\omega_0}) \ ,$$

where $\Pi(x)$ equals 1 if $| \ x \ |< \frac{1}{2}$ and equals 0 otherwise. We assume here that $\omega_0 < \pi$.

Let $y[n]$ denote the result of passing $x[n]$ through this filter. Let $y_b[n]$ be the decimation of $y[n]$ to even times, i.e.

$$y_b[n] = y[2n] \ , \quad n \in \mathbf{Z} \ .$$

Let $x_b[n]$ denote the decimation of $x[n]$ to even times, i.e.

$$x_b[n] = x[2n] \ , \quad n \in \mathbf{Z} \ ,$$

and let $z[n]$ denote the result of passing $x_b[n]$ through the filter.

Find the conditions on ω_M and ω_0 under which $z[n] = y_b[n]$ for all $n \in \mathbf{Z}$.

7. *(5 points)*

Let $x_1(t)$ have Laplace transform

$$X_1(s) = \frac{s-1}{s+2} \ , \quad Re(s) > -2 \ ,$$

and let $x_2(t)$ have Laplace transform

$$X_2(s) = \frac{s+2}{s^2+1} \ , \quad Re(s) < 0 \ .$$

Let

$$x_3(t) = x_1(t) * x_2(t) \ ,$$

denote the convolution of $x_1(t)$ and $x_2(t)$. Find the Laplace transform $X_3(s)$ of $x_3(t)$ (together with its region of convergence !).

8. *(15 points)*

In the following problem, on the next page, every correct answer with a correct reason gets 3 points, every correct answer with a wrong reason gets 1 point, and every wrong answer gets 0 points.

Match the pole/zero plots (a)-(e) with the corresponding magnitude responses (1)-(5). In each case, provide a brief justification.