EECS 120 **Fall 2004**

Homework 3 : Due by 8 p.m. on Monday September 27

1. Problem 3.37 on pp. 260 -261 of OWN.
2. Problem 3.48 on pg. 265 of OWN.
3. Problem 3.58 on pg. 270 of OWN.
4. Problem 3.62 on pg. 272 of OWN.
5. Consider the constant-coefficient difference equation

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k] \ , \tag{1}$$

where we assume that $a_0 \neq 0$. The steady state output corresponding to the input $x[n] = e^{j\omega n}$ is $y[n] = H(e^{j\omega})e^{j\omega n}$, where

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n} \ .$$

is the frequency response of the discrete time LTI system defined by the difference equation. Here $h[n]$ denotes the impulse response of this LTI system, i.e. we have

$$\sum_{k=0}^{N} a_k h[n-k] = \sum_{k=0}^{M} b_k \delta[n-k] \ ,$$

from the condition of initial rest. Recall that the steady state output corresponding to any input $x[n]$ would then be

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] \ .$$

Let a be a $1 \times (N+1)$ array in MATLAB whose i-th entry is the coefficient a_{i-1}, $1 \leq i \leq N+1$. Let b be a $1 \times (M+1)$ array in MATLAB whose i-th entry is the coefficient b_{i-1}, $1 \leq i \leq M+1$.

The MATLAB command [**H omega**] = **freqz(b, a, N)** computes the frequency response $H(e^{j\omega})$ at the frequencies $\omega_k = \frac{k}{N}\pi$, $0 \leq k \leq N-1$. *omega* now becomes a $1 \times N$ array whose k-th coordinate is $\frac{k-1}{N}\pi$, $1 \leq k \leq N$, and H becomes a $1 \times N$ array whose k-th coordinate is $H(e^{j\pi\frac{k-1}{N}})$, $1 \leq k \leq N$. Note that the entries of the array H are complex numbers !

The MATLAB command [**H omega**] = **freqz(b, a, N, 'whole')** computes the frequency response $H(e^{j\omega})$ at the frequencies $\omega_k = \frac{k}{N}2\pi$, $0 \leq k \leq N-1$. *omega* now becomes a $1 \times N$ array whose k-th coordinate is $\frac{k-1}{N}2\pi$, $1 \leq k \leq N$, and H becomes a

$1 \times N$ array whose k-th coordinate is $H(e^{j2\pi\frac{k-1}{N}})$, $1 \leq k \leq N$. Note that the entries of the array H are complex numbers !

For a complex number x the MATLAB command **y = abs(x)** returns the absolute value of x in the variable y. Thus **G = abs(H)** would return an array G whose entries are the absolute values of the corresponding entries in the array H. The MATLAB command **theta = angle(x)** returns in *theta* the angle (in radians) of the complex number x.

The MATLAB command **plot(t, f)** can be used to plot the entries in a $1 \times N$ array against the values in a $1 \times N$ array t.

(a) Consider the constant coefficient difference equation

$$y[n] = \frac{1}{3}x[n] + x[n-1] - \frac{1}{2}x[n-2] \ .$$

Give magnitude and phase plots of the frequency response of the corresponding discrete time LTI system over the interval $(-\pi, \pi]$ with a total resolution of 200 points, *with the frequency axis properly labelled.* Note : be careful, you will also need to worry about properly shifting the entries of the array H returned by MATLAB !

(b) Do the same for the constant coefficient difference equation

$$5y[n] + y[n-2] = x[n] + 3x[n-1]$$

(c) Do the same for the constant coefficient difference equation

$$2y[n] + 12y[n-1] + 24y[n-2] + 16y[n-3] = x[n-5]$$

Do NOT submit any M-files. You simply need to submit a total of six plots : a magnitude plot and a phase plot for each of three cases.