EECS 120 **Fall 2004**

Homework 4 : Due by 8 p.m. on Monday October 4

1. Problem 4.7 on pg. 335 of OWN.
2. Parts (a), (b), (c), and (j) of Problem 4.21 on pg. 338 of OWN.
3. Problem 4.36 on pg. 346 of OWN.
4. (a) Read Problem 4.45 on pg. 349 and make sure you understand how to do it. You *need not* submit a written answer to this problem. The purpose of this problem is for you to become familiar with the term *energy density spectrum*.

   (b) Problem 4.46 on pp. 349 -350 of OWN.
5. Consider the linear constant-coefficient differential equation

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t) \ , \tag{1}$$

where we assume that $a_N \neq 0$. Let $h(t)$ denote the impulse response of the system. The equation (1) defines a causal LTI continuous time system, with the output $y(t)$ corresponding to input $x(t)$ being given by $y(t) = x(t) * h(t)$ (convolution). Taking Fourier transforms, we have

$$Y(j\omega) = H(j\omega)X(j\omega) \ ,$$

where

$$H(j\omega) = \frac{b_M(j\omega)^M + b_{M-1}(j\omega)^{M-1} + \ldots + b_0}{a_N(j\omega)^N + a_{N-1}(j\omega)^{N-1} + \ldots + a_0} \ ,$$

is called the *frequency response* of the corresponding LTI system.

Let a be a $1 \times (N+1)$ array in MATLAB whose i-th entry is the coefficient a_{N-i+1}, $1 \leq i \leq N+1$. Let b be a $1 \times (M+1)$ array in MATLAB whose i-th entry is the coefficient b_{M-i+1}, $1 \leq i \leq M+1$.

The MATLAB command **freqs**(b, a) returns plots of the magnitude and phase of the frequency response of the LTI system corresponding to equation (1). The magnitude is plotted on a log-log plot, and the phase is plotted on a linear scale with the frequency axis being on a logarithmic scale. MATLAB automatically chooses the ranges to use for these plots.

If one wishes to choose the ranges to use, one can use the following sequence of commands (as an example) : $w =$ **linspace**$(0, 10 * pi)$ will define an array of 100 uniformly spaced samples from 0 through 9.9π. $H =$ **freqs**(b, a, w) will return the values of the frequency response at the frequencies in the array w. Thus H is now an array of 100 complex numbers. One can define $G =$**abs**(H) to get the magnitudes and $\theta =$**angle**(H) to get the angles of the entries in H, and then use **plot**(w, G) to plot G on a linear scale or **semilogx**(w, G) or **semilogy**(w, G), or **loglog**(w, G) etc. if one wants semilogarithmic or log-log plots (look up the definitions of these MATLAB commands).

(a) Consider the constant-coefficient differential equation

$$\frac{d^2}{dt^2}y(t) + 2\frac{d}{dt}y(t) + \frac{1}{2}y(t) = x(t)\ .$$

Plot the magnitude and the phase of its frequency response using the MATLAB command **freqs**.

(b) Consider the constant-coefficient differential equation

$$\frac{d^2}{dt^2}y(t) - y(t) = x(t)\ .$$

Plot the magnitude and the phase of its frequency response using the MATLAB command **freqs**.

(c) Consider the constant-coefficient differential equation

$$\frac{d^2}{dt^2}y(t) + y(t) = x(t)\ .$$

i. Plot the magnitude of the frequency response on a log-log scale and the phase on a linear-log scale, using 100 points, over the interval of frequencies $[0, 10]$

ii. Plot the magnitude of the frequency response on a log-log scale and the phase on a linear-log scale, using 100 points, over the interval of frequencies $[0.9, 1.1]$

iii. Plot the magnitude of the frequency response on a log-log scale and the phase on a linear-log scale, using 100 points, over the interval of frequencies $[0.99, 1.01]$

iv. Plot the magnitude of the frequency response on a log-log scale and the phase on a linear-log scale, using 100 points, over the interval of frequencies $[0.999, 1.001]$

Do NOT submit any M-files. You simply need to submit a total of six pairs of plots : a magnitude plot and a phase plot for each of six cases.