EECS 120 **Fall 2004**

Homework 9 : Due by 8 p.m. on Monday November 15

1. Problem 9.35 on pp. 729 -730 of OWN.
2. Parts (i) and (iii) of part (b) of Problem 6.29 on pp. 491 -492 of OWN.
3. Problem 9.50 on pg. 735 of OWN.
4. Parts (b), (d), and (g) of Problem 11.24 on pg. 872 of OWN.
5. Consider the linear constant-coefficient differential equation

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) = \sum_{k=0}^{M} b_k \frac{d^k}{dt^k} x(t) \ , \tag{1}$$

where we assume that $a_N \neq 0$.

Let a be a $1 \times (N+1)$ array in MATLAB whose i-th entry is the coefficient a_{N-i+1}, $1 \leq i \leq N+1$. Let b be a $1 \times (M+1)$ array in MATLAB whose i-th entry is the coefficient b_{M-i+1}, $1 \leq i \leq M+1$.

The MATLAB command **freqs**(b, a) returns plots of the magnitude and phase of the function

$$H(j\omega) = \frac{b_M (j\omega)^M + b_{M-1}(j\omega)^{M-1} + \ldots + b_0}{a_N (j\omega)^N + a_{N-1}(j\omega)^{N-1} + \ldots + a_0} \ , \tag{2}$$

The magnitude is plotted on a log-log plot, and the phase is plotted on a linear scale with the frequency axis being on a logarithmic scale. MATLAB automatically chooses the ranges to use for these plots. If one wishes to choose the ranges to use, one can use the following sequence of commands (as an example) : $w =$ **linspace**$(0, 10 * pi)$ will define an array of 100 uniformly spaced samples from 0 through 9.9π. $H =$ **freqs**(b, a, w) will return the values of the frequency response at the frequencies in the array w. Thus H is now an array of 100 complex numbers. One can define $G =$**abs**(H) to get the magnitudes and $\theta =$**angle**(H) to get the angles of the entries in H, and then use **plot**(w, G) to plot G on a linear scale or **semilogx**(w, G) or **semilogy**(w, G), or **loglog**(w, G) etc. if one wants semilogarithmic or log-log plots. This was explained in Homework 4, and is being repeated here for convenience.

If we consider the solutions of the differential equation (1) from the condition of initial rest, it defines a linear time invariant system. Let $h(t)$ denote the impulse response of the system. If the system is stable, i.e. if all the roots of $a_N s^N + a_{N-1} s^{N-1} + \ldots + a_0$ are strictly in the left half plane, then the Fourier transform of $h(t)$ is well defined and equals $H(j\omega)$ as defined above. $H(j\omega)$ as defined in (2) can then reasonably be called the *frequency response* of the corresponding LTI system. Note that in this case the rational function

$$H(s) = \frac{b_M (s)^M + b_{M-1}(s)^{M-1} + \ldots + b_0}{a_N (s)^N + a_{N-1}(s)^{N-1} + \ldots + a_0} \ ,$$

with region of convergence being the one that is unbounded to the right, would be the Laplace transform of $h(t)$, and would have the imaginary axis in the region of convergence.

More generally, for every choice of the region of convergence for the rational function $H(s)$, it becomes the Laplace transform of some time function. Treating this time function as the impulse response then defines an LTI system asssociated to the differential equation (1). Only at most one of the various LTI systems so defined will have the imaginary axis in the region of convergence of the Laplace transform. It is for that system that it would make sense to call $H(j\omega)$, as defined in (2), the frequency response of the system.

(a) Consider the constant-coefficient differential equation

$$\frac{d^2}{dt^2}y(t) + 2\frac{d}{dt}y(t) + \frac{1}{2}y(t) = x(t) \ .$$

This is identical to the one you saw in part (a) of Problem 5 of Homework 4.

i. Verify that the causal LTI system defined by this differential equation is stable.

ii. Plot the magnitude and the phase of its frequency response using the MATLAB command **freqs**. These are Bode plots.

iii. On the Bode plots you get, sketch the straight line approximations by hand.

(b) Consider the constant-coefficient differential equation

$$\frac{d^2}{dt^2}y(t) - y(t) = x(t) \ .$$

This is identical to the one you saw in part (b) of Problem 5 of Homework 4.

i. Verify that the causal LTI system defined by this differential equation is *not* stable.

ii. Find the impulse response of an LTI system determined by this differential equation for which it makes sense to call $H(j\omega)$, as defined in (2), the frequency response of the system.
Hint : This impulse response will be neither causal nor anticausal.

(c) Consider the constant-coefficient differential equation

$$\frac{d^2}{dt^2}y(t) + y(t) = x(t) \ .$$

This is identical to the one you saw in part (c) of Problem 5 of Homework 4.

i. Verify that the causal LTI system defined by this differential equation is *not* stable.

ii. Is there any LTI system defined by this differential equation for which it makes sense to call $H(j\omega)$, as defined in (2), the frequency response of the system ?

(d) Consider the constant-coefficient differential equation

$$\frac{d^4}{dt^4}y(t) + 7\frac{d^3}{dt^3}y(t) + 18\frac{d^2}{dt^2}y(t) + 21\frac{d}{dt}y(t) + 9y(t) = \frac{d}{dt}x(t) - x(t) \ .$$

i. Verify that the causal LTI system defined by this differential equation is stable.
ii. Plot the magnitude and the phase of its frequency response using the MATLAB command **freqs**. These are Bode plots.
iii. On the magnitude Bode plot you get, sketch the straight line approximation by hand.