Notes on Intersymbol interference

We discussed amplitude modulation of a baseband modulating signal by a pulse train carrier. Specifically, we considered the carrier

$$c(t) = \sum_{n=-\infty}^{\infty} \Pi(\frac{t-nT}{\Delta}) \ ,$$

where

$$\Pi(t) = \begin{cases} 1 & \text{if } \mid t \mid < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \ .$$

If the modulating signal is $x(t)$, we would create $x(t)c(t)$.

Pulse amplitude modulation is the term used for the modulation technique where instead of creating $x(t)c(t)$ as above, we create

$$y(t) = \sum_{n=-\infty}^{\infty} x(nT)\Pi(\frac{t-\frac{\Delta}{2}-nT}{\Delta}) \ .$$

This signal also moves the energy of the baseband modulating signal to around the carrier frequency (and its harmonics). It may be thought of as being created by periodically sampling the modulating signal and then using the samples to adjust the respective heights of a pulse train. If the modulating signal is bandlimited to $(-\omega_M, \omega_M)$ where $\omega_c > 2\omega_M$, then, in principle, it can be exactly recovered from $y(t)$ [1]. One simple way to do this is to sample $y(t)$ at the times $nT + \frac{\Delta}{2}$. Note that

$$y(nT + \frac{\Delta}{2}) = x(nT) \ .$$

In reality the communication channel from transmitter to the receiver is not ideal, so the received signal $r(t)$ is different from $y(t)$. Suppose we can write

$$r(t) = y(t) * h(t) \ ,$$

[1] The modulating signal can also be recovered if what is transmitted is $y(t)$ filtered through an ideal bandpass filter which passes $\pm[\omega_c - \omega_M, \omega_c + \omega_M]$. We will assume for this discussion that such filtering is not being done.

i.e. the channel is modeled as an LTI system with impulse response $h(t)$. Then the samples $r(nT + \frac{\Delta}{2})$ no longer equal the respective $x(nT)$. In fact, each of these samples could potentially have a contribution from several of the samples of the modulating signal. This phenomenon is called *intersymbol interference* (ISI).

A commonly used idea to overcome ISI is to work with a pulse shape other than the rectangular pulse $\Pi(t)$. Thus, the transmitter might actually transmit the signal

$$y(t) = \sum_{n=-\infty}^{\infty} x(nT) p(\frac{t - \frac{\Delta}{2} - nT}{\Delta}) \ ,$$

where $p(t)$ is a signal chosen such that, if we define $z(t) = p(t) * h(t)$, we have

$$z(nT) = \begin{cases} \kappa & \text{if } n = 0 \\ 0 & \text{if } n \neq 0 \end{cases} \ , \tag{1}$$

for some constant κ. Note that the received signal, with the same LTI channel model as before, would be

$$r(t) = \sum_{n=-\infty}^{\infty} x(nT) z(\frac{t - \frac{\Delta}{2} - nT}{\Delta}) \ .$$

We would then have

$$r(nT + \frac{\Delta}{2}) = \sum_{k=-\infty}^{\infty} x(kT) z(\frac{nT - kT}{\Delta}) = \kappa x(nT) \ ,$$

so we can recover the modulating signal at the receiver.

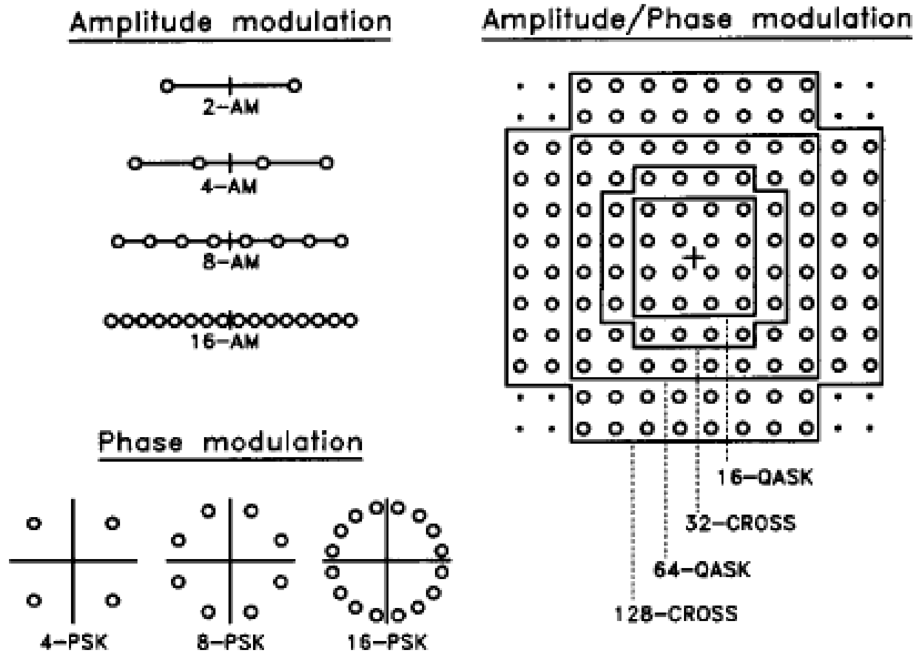
Typically, the impulse response of the channel is unknown because it varies (it must vary on a slow time scale relative to the sampling interval for the idea of using a different pulse shape to make sense). What is then often done is that the receiver estimates the channel impulse response. This can be done, for instance, by having the transmitter send a *training signal* (this of course reduces the efficiency of the overall communication scheme), which is known to the receiver before hand, and which can therefore be used to estimate the impulse response of the channel.

If one thinks a little bit harder about the suggestion in the preceding paragraph, one realizes that there is still something missing from this approach. Indeed, since the pulse shape $p(t)$ has to be chosen so that $z(t) = p(t) * h(t)$ has the property (1), the transmitter would in principle have to keep adjusting $p(t)$ as $h(t)$ varies. This might require feedback of the estimates of $h(t)$ from the receiver to the transmitter. What is often done is something different. The receiver uses a filter, called the *channel equalizer*, whose impulse response it can adjust (by means of adjustable taps in a filter implementation). This filter is in series with the channel. Let $h_e(t)$ denote the impulse response of the equalizer. As the receiver changes its estimates for $h(t)$ it adjusts $h_e(t)$ so that the convolution of the estimated channel impulse response with the current equalizer impulse response is always kept equal to a target impulse response (as best as possible). The pulse shape used by the transmitter is actually designed for this target impulse response, so it never needs to be changed.

Rudiments of digital communication schemes

Typically, the samples from the modulating signal are not directly used to adjust the amplitude of a pulse. Rather, they are *quantized*, i.e. each real valued sample is actually represented by some fixed finite number of bits. The quantized samples are then subject to *data compression*, whereby the total number of bits that need to be sent per sample is reduced. The compressed bits are then (perhaps after introducing some controlled redundancy through the use of *error control codes*) typically grouped and the groups are used to select points from a *constellation* of (typically) complex numbers, called *symbols*. For instance, if one uses groups of 4 bits at a time, one could use each such four-tuple to select one of a collection of 16 constellation points. The constellation point (a, b) (or, equivalently, $a + jb$) is transmitted over one *symbol interval* of length T as the signal $a\cos(\omega_c t) + b\sin(\omega_c t)$. Each successive group of bits thereby determines what the transmitter sends out in each successive symbol interval. All these steps are reversed at the receiver.

Some typical examples of constellations are shown on the next page.


Amplitude modulation
2-AM
4-AM
8-AM
16-AM
Phase modulation
4-PSK
8-PSK
16-PSK
Amplitude/Phase modulation
16-QASK
32-CROSS
64-QASK
128-CROSS