Discussion Review Midterm 1
Week of September 29 - October 5, 2005

Problem 1 *(Short questions.)* 35 Points (5 Points each)

(a) For the following system with input $x[n]$ and output $y[n]$, determine whether the statements are true or false.

$$y[n] = \frac{1}{1 + x[2n]}$$

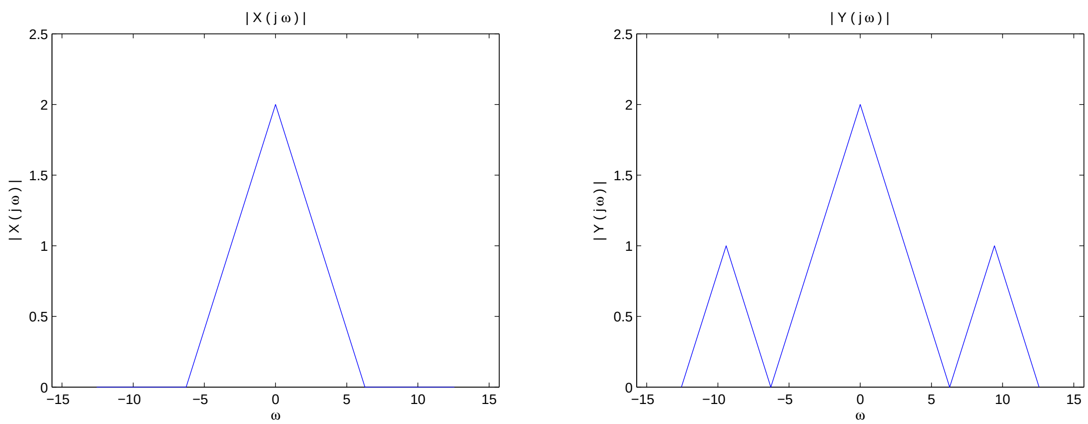
T F the system is linear
T F the system is time-invariant
T F the system is memoryless
T F the system is stable
T F the system is causal

(b) A discrete-time system has input $x(t)$ and output $y(t)$ such that

$$y[n] = x[n] - y[n-1]$$

Is the system stable? If it is stable, give a short proof. If not, give a counterexample.

(c) A signal $x(t)$ is the input to an unknown system. The signal $y(t)$ is the output. The magnitudes of the Fourier transfoms of the input and output signals are given below.



Determine if the following statement is true, or False, or Not Enough Information? Explain your answer, briefly (approx 1-3 sentences).

T F NEI The system is both linear and time-invariant.

(d)

$$\text{Given } x(t) = \begin{cases} 1, & 0 \le t < \frac{1}{2}, \\ -1, & \frac{1}{2} \le t < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Plot $x(\frac{t}{4} - 3)$. Label your axes clearly and carefully!

(e) A discrete-time LTI system with input $x[n]$ and output $y[n]$ is described by the following constant coefficient difference equation:

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = x[n]$$

If $x[n] = \cos(\pi n)$, what is $y[n]$?

(f) The signals $x_1(t)$ and $x_2(t)$ are defined below.

$$x_1(t) = \begin{cases} 1 - |t|, & |t| < 1 \\ 0, & |t| \ge 1 \end{cases}$$

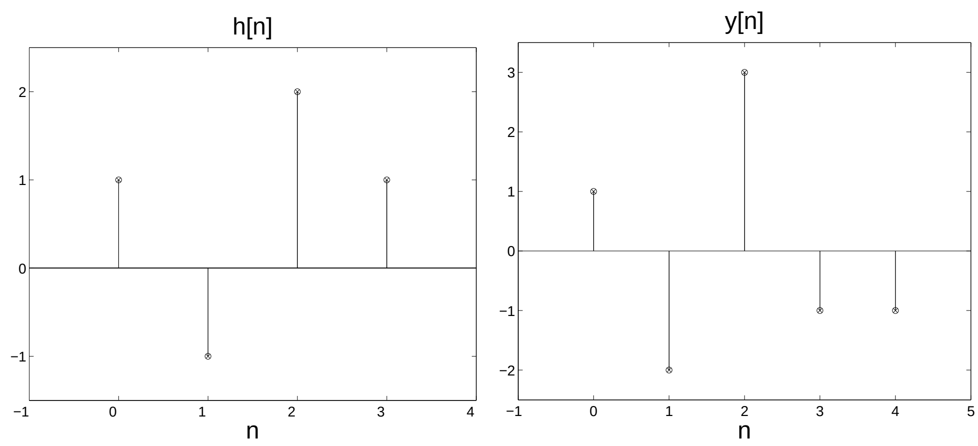
$$x_2(t) = \delta(t+2) + 2\delta(t-2)$$

Plot the convolution of the two signals, $y(t) = x_1(t) * x_2(t)$, clearly labeling the time axis and amplitudes.

(g) A given discrete-time LTI system has impulse response $h[n]$, input $x[n]$, and output $y[n]$. $h[n]$ and $y[n]$ are given below.

$$h[n] = \delta[n] - \delta[n-1] + 2\delta[n-2] + \delta[n-3]$$

$$y[n] = \delta[n] - 2\delta[n-1] + 3\delta[n-2] - \delta[n-3] - \delta[n-4]$$



Given that $x[n]$ is causal, graph $x[n]$, for $-1 \le n \le 5$, carefully labeling the time axis and amplitudes.

Problem 2 20 Points

(a) *(i)* (5 Points)

Find the Fourier transform $X(j\omega)$ of

$$x(t) = \begin{cases} 4 - |t|, & |t| \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

(ii) (7 Points)

The signal $y(t)$ is defined below

$$y(t) = \begin{cases} t + 8, & -8 \leq t < -4, \\ 4, & -4 \leq t < 4, \\ 8 - t, & 4 \leq t < 8, \\ 0, & \text{otherwise.} \end{cases}$$

Find the Fourier transform $Y(j\omega)$ of $y(t)$.

(b) (7 Points)

Compute the following integral:

$$\int_{-\infty}^{\infty} \left(\frac{\sin(7\tau)}{\pi\tau}\right) \left(\frac{\sin(3(\frac{\pi}{4} - \tau))}{\pi(\frac{\pi}{4} - \tau)}\right) \, d\tau$$

Problem 3 10 Points

Let $x[n]$ be a periodic sequence with period N. Assume $N = 3K$ for some integer K. Let a_k denote the discrete time Fourier series coefficients of $x[n]$. If $a_k = 0$ when k is not a multiple of 3, show that $x[n]$ must also be periodic with period K.