Problem 1 (Short questions.)  

(a) For the following system with input $x[n]$ and output $y[n]$, determine whether the statements are true or false.

\[ y[n] = \frac{1}{1 + x[2n]} \]

T  F  the system is linear  
T  F  the system is time-invariant  
T  F  the system is memoryless  
T  F  the system is stable  
T  F  the system is causal

(b) A discrete-time system has input $x(t)$ and output $y(t)$ such that

\[ y[n] = x[n] - y[n - 1] \]

Is the system stable? If it is stable, give a short proof. If not, give a counterexample.

(c) A signal $x(t)$ is the input to an unknown system. The signal $y(t)$ is the output. The magnitudes of the Fourier transforms of the input and output signals are given below.

![Fourier Transform Graphs]

Determine if the following statement is true, or False, or Not Enough Information? Explain your answer, briefly (approx 1-3 sentences).
(d) Given \( x(t) = \begin{cases} 
1, & 0 \leq t < \frac{1}{2}, \\
-1, & \frac{1}{2} \leq t < 1, \\
0, & \text{otherwise}. 
\end{cases} \)

Plot \( x\left(\frac{t}{4} - 3\right) \). Label your axes clearly and carefully!

(e) A discrete-time LTI system with input \( x[n] \) and output \( y[n] \) is described by the following constant coefficient difference equation:

\[
y[n] - \frac{5}{6} y[n-1] + \frac{1}{6} y[n-2] = x[n]
\]

If \( x[n] = \cos(\pi n) \), what is \( y[n] \)?

(f) The signals \( x_1(t) \) and \( x_2(t) \) are defined below.

\[
x_1(t) = \begin{cases} 
1 - |t|, & |t| < 1 \\
0, & |t| \geq 1 
\end{cases}
\]

\[
x_2(t) = \delta(t + 2) + 2\delta(t - 2)
\]

Plot the convolution of the two signals, \( y(t) = x_1(t) \ast x_2(t) \), clearly labeling the time axis and amplitudes.

(g) A given discrete-time LTI system has impulse response \( h[n] \), input \( x[n] \), and output \( y[n] \). \( h[n] \) and \( y[n] \) are given below.

\[
h[n] = \delta[n] - \delta[n - 1] + 2\delta[n - 2] + \delta[n - 3] \\
y[n] = \delta[n] - 2\delta[n - 1] + 3\delta[n - 2] - \delta[n - 3] - \delta[n - 4]
\]

Given that \( x[n] \) is causal, graph \( x[n] \), for \(-1 \leq n \leq 5\), carefully labeling the time axis and amplitudes.
**Problem 2**

20 Points

(a) (i) (5 Points)

Find the Fourier transform \( X(j\omega) \) of

\[
x(t) = \begin{cases} 
4 - |t|, & |t| \leq 4, \\
0, & \text{otherwise.}
\end{cases}
\]

(ii) (7 Points)

The signal \( y(t) \) is defined below

\[
y(t) = \begin{cases} 
t + 8, & -8 \leq t < -4, \\
4, & -4 \leq t < 4, \\
8 - t, & 4 \leq t < 8, \\
0, & \text{otherwise.}
\end{cases}
\]

Find the Fourier transform \( Y(j\omega) \) of \( y(t) \).

(b) (7 Points)

Compute the following integral:

\[
\int_{-\infty}^{\infty} \left( \frac{\sin(7\tau)}{\pi \tau} \right) \left( \frac{\sin(3(\frac{\tau}{4} - \tau))}{\pi (\frac{\tau}{4} - \tau)} \right) \, d\tau
\]

**Problem 3**

10 Points

Let \( x[n] \) be a periodic sequence with period \( N \). Assume \( N = 3K \) for some integer \( K \). Let \( a_k \) denote the discrete time Fourier series coefficients of \( x[n] \). If \( a_k = 0 \) when \( k \) is not a multiple of 3, show that \( x[n] \) must also be periodic with period \( K \).