## Quiz 1 Answers

## Name:

SID:
Formulas that may be of use:

- Continuous time Fourier series:
$x(t)=\sum_{k=-\infty}^{+\infty} a_{k} e^{j k \omega_{0} t}=\sum_{k=-\infty}^{+\infty} a_{k} e^{j k(2 \pi / T) t}$,
$a_{k}=\frac{1}{T} \int_{T} x(t) e^{-j k \omega_{0} t} d t=\frac{1}{T} \int_{T} x(t) e^{-j k(2 \pi / T) t} d t$
- Discrete time Fourier series:
$x[n]=\sum_{k=<N>} a_{k} e^{j k \omega_{0} n}=\sum_{k=<N>} a_{k} e^{j k(2 \pi / N) n}$,
$a_{k}=\frac{1}{N} \sum_{n=<N>} x[n] e^{-j k \omega_{0} n}=\frac{1}{N} \sum_{n=<N>} x[n] e^{-j k(2 \pi / N) n}$

Problem 1 (System Properties)

1. $y(t)=\int_{-\infty}^{2 t} x(\tau) d \tau$
2. $y[n]=x[n] \sum_{k=-\infty}^{\infty} \delta[n-3 k]$

Determine whether the above systems are

- Memoryless (2)
- Linear $(1,2)$
- Time invariant
- Causal (2)
- Stable (2)


## Problem 2 (Convolution)

If $y[n]=h[n] * x[n]$, where $h[n]$ and $x[n]$ are shown below in Figure 1.
(a) Find the value of $n$ at which $y[n]$ is maximum.
(b) What is the maximum value of $y[n]$.

Answer: (a)


(b) $\square$


Figure 1: Problem 2

## Problem 3 (Frequency Response)

$y(t)$ is the output of an LTI system with frequency response $H(j \omega)=\frac{1}{j \omega}$ when the input $x(t)=$ $2 e^{j 2 t}-\sin (-\pi t)$ is fed into the system. Determine $y(0), y\left(\frac{1}{2}\right), y(100)$.

Answer $y(0)=\frac{-1}{\pi}-j$
Answer $y\left(\frac{1}{2}\right)=-j e^{j}$
Answer $y(100)=-j e^{j 200}-\frac{1}{\pi}$

Problem 4 (DTFS) Let $a_{k}$ be the Fourier series coefficients. Find the value of $a_{0}$ for the following signals.
(a) $x[n]$ is periodic with period 4 and $x[n]=1-\sin \frac{\pi n}{4}$ for $0 \leq n \leq 3$.

Answer $a_{0}=\frac{1}{4}(3-\sqrt{2})$
(b) $x[n]$ is periodic with period 12 and $x[n]=1-\sin \frac{\pi n}{4}$ for $0 \leq n \leq 11$.

Answer $a_{0}=\frac{1}{12}(11-\sqrt{2})$

