Quiz 1 Answers

Name:

SID:

Formulas that may be of use:

- Continuous time Fourier series: $\begin{aligned} x(t) &= \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t} , \\ a_k &= \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt \end{aligned}$
- Discrete time Fourier series: $\begin{aligned} x[n] &= \sum_{k=<N>} a_k e^{jk\omega_0 n} = \sum_{k=<N>} a_k e^{jk(2\pi/N)n}, \\ a_k &= \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=<N>} x[n] e^{-jk(2\pi/N)n} \end{aligned}$

Problem 1 (System Properties)

1. $y(t) = \int_{-\infty}^{2t} x(\tau) d\tau$ 2. $y[n] = x[n] \sum_{k=-\infty}^{\infty} \delta[n-3k]$

Determine whether the above systems are

- Memoryless (2)
- Linear (1,2)
- Time invariant
- Causal (2)
- Stable (2)

Problem 2 (Convolution)

If y[n] = h[n] * x[n], where h[n] and x[n] are shown below in Figure 1.

(a) Find the value of n at which y[n] is maximum.

(b) What is the maximum value of y[n].

Answer: (a) -7 (b) 6



Figure 1: Problem 2

Problem 3 (Frequency Response)

y(t) is the output of an LTI system with frequency response $H(j\omega) = \frac{1}{j\omega}$ when the input $x(t) = 2e^{j2t} - \sin(-\pi t)$ is fed into the system. Determine $y(0), y(\frac{1}{2}), y(100)$.



Problem 4 (*DTFS*) Let a_k be the Fourier series coefficients. Find the value of a_0 for the following signals.

(a) x[n] is periodic with period 4 and $x[n] = 1 - \sin \frac{\pi n}{4}$ for $0 \le n \le 3$.

Answer $a_0 = \frac{1}{4}(3 - \sqrt{2})$

(b) x[n] is periodic with period 12 and $x[n] = 1 - \sin \frac{\pi n}{4}$ for $0 \le n \le 11$.

Answer $a_0 = \frac{1}{12}(11 - \sqrt{2})$