EE 120
Fall 07

Discussion 12

Announcements

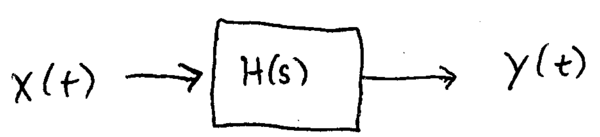
Quiz - Thursday
Midterm 2 - Thursday, Dec. 6, evening
Gastpar's Supplemental notes on Bode Diagrams on web

Agenda

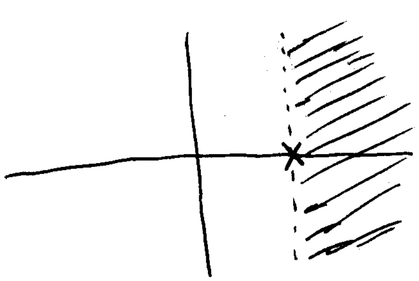
- Feedback Control
- Bode diagrams

Feedback

goal: stabilize an unstable system



$H(s) = \frac{1}{s-2}$ and $h(t)$ is a causal system



$\Rightarrow$ system is unstable

Solution 1: pole-zero cancellation

$$x(t) \longrightarrow \underset{\text{"plant"}}{\boxed{H(s)}} \longrightarrow \underset{\text{"controller"}}{\boxed{G(s)}} \longrightarrow y(t)$$

$$T(s) = \frac{Y(s)}{X(s)} = H(s) \cdot G(s)$$

design controller so that $G(s) = \dfrac{s-2}{s+3}$

$$\Rightarrow \quad T(s) = \frac{1}{s-2} \cdot \frac{s-2}{s+3} = \frac{1}{s+3}$$

$\Rightarrow$ stable system

what if $G(s) = \dfrac{s-2.03}{s+3.01}$

$$T(s) = \frac{s-2.03}{(s-2)(s+3.01)}$$

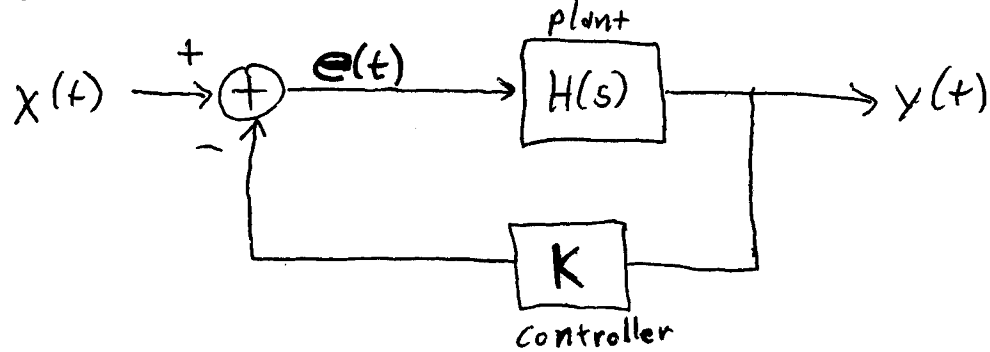
→ no pole-zero cancellation

→ system is unstable

→ mention $G(s)$ is built out of analog components, etc.

→ "all or nothing" approach

Solution 2: Feedback loop



K: adjustable gain

$$\begin{cases} Y(s) = E(s) \cdot H(s) \\ E(s) = X(s) - K \cdot Y(s) \end{cases}$$

$$Y(s) = H(s)\left[X(s) - K \cdot Y(s)\right]$$

$$Y(s) \cdot \left[1 + K \cdot H(s)\right] = X(s) \cdot H(s)$$

$$T(s) = \frac{Y(s)}{X(s)} = \frac{H(s)}{1 + K \cdot H(s)}$$

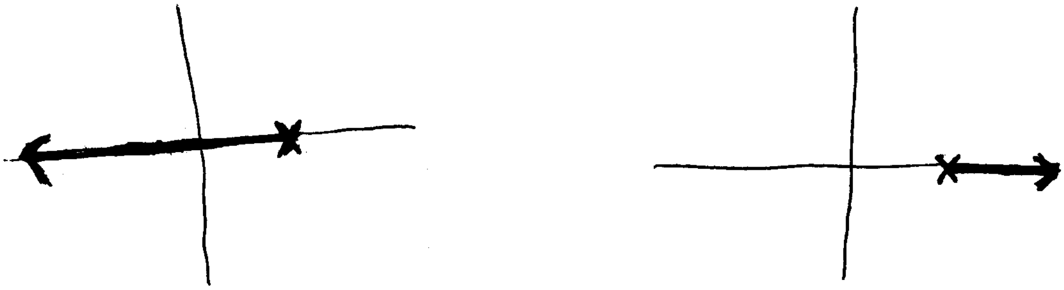
in our problem, $H(s) = \frac{1}{s-2}$

$$T(s) = \frac{\frac{1}{s-2}}{1 + K \cdot \frac{1}{s-2}} = \frac{1}{s-2+K} = \frac{1}{s-(2-K)}$$

$\Rightarrow$ one pole at $s = 2-K$

$\Rightarrow$ system is stable if $2-K < 0$

i.e., if $K > 2$



$K > 0$ $\qquad\qquad$ $K < 0$

"Root Locus"

"Root" ← roots of denominator polynomial

"Locus" ← "set of all"

design controller so that $K = 3$

$$\Rightarrow T(s) = \frac{1}{s+1}$$

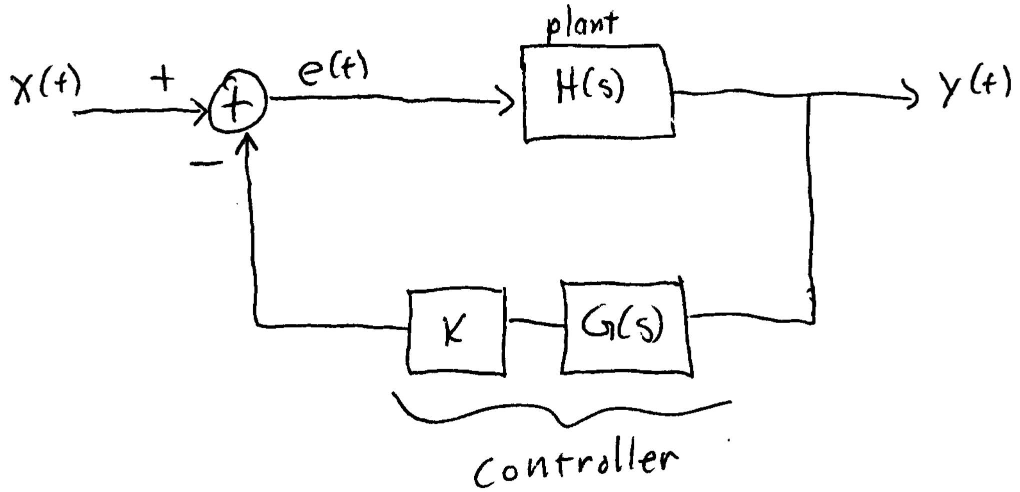
what if $K = 2.98$?

$$\Rightarrow T(s) = \frac{1}{s + 0.98}$$

$\Rightarrow$ system is still stable

$\Rightarrow$ robust to small defects/errors/etc.

General feedback system



$G(s)$: fixed

K: adjustable gain

$$Y(s) = H(s) \cdot E(s)$$

$$E(s) = X(s) - K \cdot G(s) \cdot H(s)$$

$$T(s) = \frac{Y(s)}{X(s)} = \frac{H(s)}{1 + K \cdot G(s) \cdot H(s)}$$

poles of $T(s)$: $\quad 1 + K \cdot H(s) \cdot G(s) = 0$

$$H(s) \cdot G(s) = \frac{-1}{K}$$

if $K = 0 \Rightarrow$ poles of $H(s) \cdot G(s)$

if $|K| = \infty \Rightarrow$ zeros of $H(s) \cdot G(s)$

$$H(s) = \frac{1}{s-1} \qquad G(s) = \frac{1}{s+3}$$

$$T(s) = \frac{\frac{1}{s-1}}{1 + K \cdot \frac{1}{s-1} \cdot \frac{1}{s+3}} = \frac{s+3}{(s-1)(s+3) + K}$$

$$= \frac{s+3}{s^2 + 2s - 3 + K} = \frac{s+3}{s^2 + 2s + (K-3)}$$

poles at

$$s = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot (K-3)}}{2}$$

$$= -1 \pm \sqrt{1 - (K-3)}$$

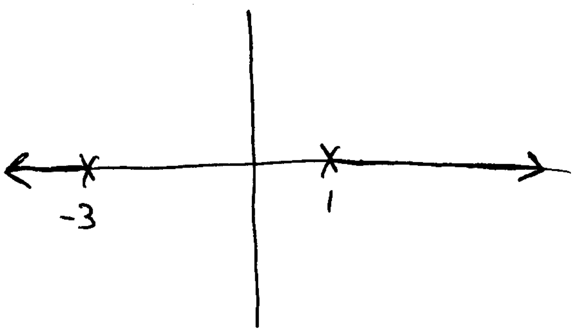
$$= -1 \pm \sqrt{4 - K}$$

$K = 0 \Rightarrow$ poles at $-3, 1$

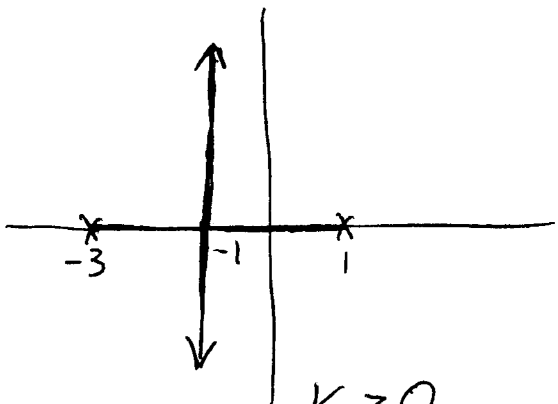
$K < 0 \Rightarrow$ poles at $-1 \pm \sqrt{4-K}$

$0 \le K \le 4 \Rightarrow$ poles at $-1 \pm \sqrt{4-K}$

$K > 4 \Rightarrow$ poles at $-1 \pm j\sqrt{K-4}$



$K < 0$



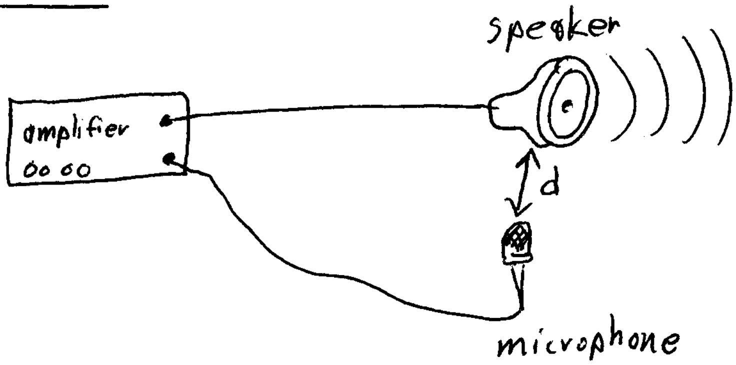
$K > 0$

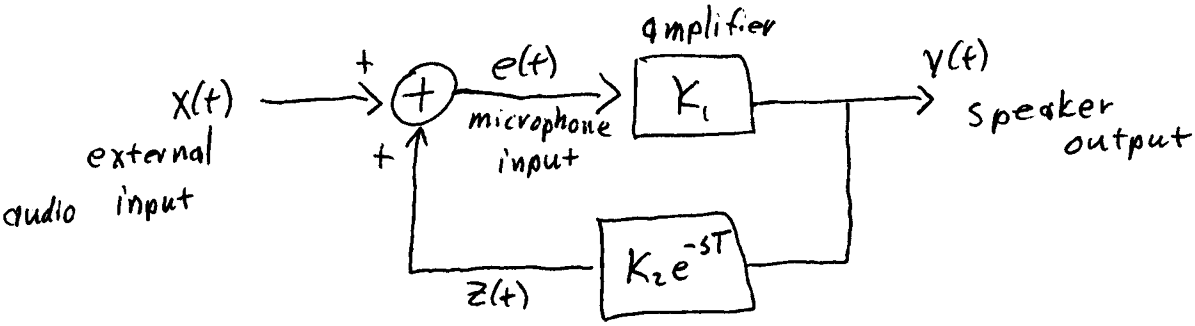
stable if $-1 + \sqrt{4-K} < 0$

$\sqrt{4-K} < 1$

$4 - K < 1$

$K > 3$





$$z(t) = K_2 \cdot y(t-T) \quad \longleftrightarrow \quad Z(s) = K_2 \cdot e^{-sT} \cdot Y(s)$$

$$Y(s) = K_1 \cdot E(s)$$

$$E(s) = X(s) + K_2 \cdot e^{-sT} \cdot Y(s)$$

$$T(s) = \frac{Y(s)}{X(s)} = \frac{K_1}{1 - K_1 K_2 e^{-sT}}$$

pole at $\quad 1 - K_1 K_2 e^{-sT} = 0$

$$e^{-sT} = \frac{1}{K_1 K_2}$$

$$-sT = \log \frac{1}{K_1 K_2}$$

$$s = \frac{-1}{T} \cdot \log\left(\frac{1}{K_1 K_2}\right)$$

if $K_1 K_2 > 1$

then $\log \frac{1}{K_1 K_2} < 0$

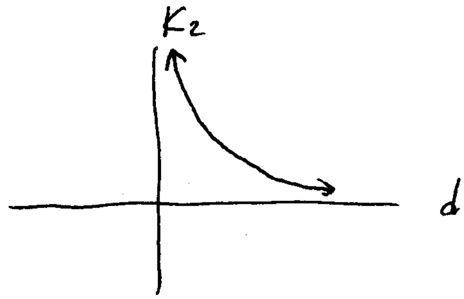
then $\frac{-1}{T} \cdot \log \frac{1}{K_1 K_2} > 0$

then system is unstable

gain K_2 can be modeled as

$$K_2 \approx \frac{c_2}{d^2} \qquad \left(\text{or } K_2 \approx \frac{c_2}{d^\alpha} \right)$$

as d gets small, K_2 increases



if d is small enough,
then $K_1 K_2 > 1$ and the system gets unstable and you hear horrible screeching sounds