1. (30 pts) DFT (Lec. 11,12,13 DFT H.O.)
Consider the signal flow diagram shown in Figure 1. For each window \( w(t) \), signal \( x(t) \), and sampling combination below, sketch \( x(t), x_w(t), x_δ(t), x'(t) \) and their magnitude spectra. Also sketch magnitude and phase for \( X[k] \) (derived from \( X'(j\omega) \)).

i. Let \( w(t) = \Pi(\frac{t}{T_o}), T_o = 8T_s, T_s = 1/3 \) sec, \( x(t) = \cos(3\pi t/2) \).

ii. Let \( w(t) = \Pi(\frac{2t}{T_o}), T_o = 8T_s, T_s = 1/3 \) sec, \( x(t) = \cos(3\pi t/2) \).

iii. Let \( w(t) = \Pi(\frac{2(t-T_o/4)}{T_o}), T_o = 8T_s, T_s = 1/3 \) sec, \( x(t) = \cos(3\pi t/2) \).

\[
\begin{align*}
&x(t) & x'(t) & \sum_\delta(t-nT_o) \\
&w(t) & & \sum_\delta(t-nT_s) \\
&x_\delta(t) & x_w(t) & \Sigma \delta(t-nT_o)
\end{align*}
\]

Fig. 1. DFT equivalent block diagram.

2. (20 pts) DFT (Lec. 11,12, DFT H.O.)
The DFTs of the signals \( x[n] = \cos(\omega_o(\frac{nT_o}{N} - \tau)) \) and \( y[n] = \cos(\omega_o \frac{nT_o}{N}) \) are calculated, with \( \omega_o = 2\pi 13.7 \), \( n = 0..255 \), \( T_o = 1 \) sec, and \( N = 256 \), as shown below for samples \( X[0]...X[31] \), and \( Y[0]...Y[31] \).

a) Using reasoning as in problem 1 above, explain the differences between the DFT of \( x[n] \) and \( X(j\omega) \), the FT of \( x(t) = \cos(\omega_o t) \).

b) \( Y[k] \) is complex. A time shift \( \tau \) was used to make \( X[k] \) pure real. Determine this value of \( \tau \), and show using the DFT analysis equation why \( X[k] \) is real.
3. (30 pts) Lec14 OW Ch. 7
(Refer to OW Fig. 7.37). The procedure for interpolation or upsampling by an integer factor \( N \) can be thought of as a cascade of two operations. The first operation, involving system A, corresponds to inserting \( N-1 \) zero-sequence values between each sequence value of \( x[n] \), such that:
\[
    x_p[n] = x_d[n/N] \quad \text{for} \ n = 0, \pm N, \pm 2N, \ldots \text{and 0 otherwise.}
\]
For exact bandlimited interpolation, \( H(e^{j\omega}) \) is an ideal low-pass filter.

a. Determine whether or not system A is linear.
b. Determine whether or not system A is time invariant.
c. For \( X_d(e^{j\omega}) \) as shown below, and with \( N = 3 \), sketch \( X_p(e^{j\omega}) \).
d. For \( N = 3 \), \( X_d(e^{j\omega}) \) as shown below, and \( H(e^{j\omega}) \) appropriately chosen for exact bandlimited interpolation, sketch \( X(e^{j\omega}) \).

\[
\begin{array}{c}
\omega = -\pi & \quad 0 & \quad \pi & \quad 2\pi \\
X_d(e^{j\omega}):
\end{array}
\]

4. (20 pts) Upsampling, Lec 14, Ch. 7
Download PS6-upsample.ipynb and music.wav from the class web page.
The bandlimited sound sample \( x[n] \) has been down sampled to 8820 Hz. Upsample back to 44.1 kHz, and use an appropriate DFT interpolation filter to create \( y[n] \) by filling in missing samples.
a) Plot the magnitude of the DFT for \( x \) and \( y \), and specify the interpolation filter \( H[k] \).
b) Plot \( x[8000 : 8200] \) and \( y[40000 : 41000] \).
c) Save and listen to the upsampled and interpolated signal. How does it compare to the original signal?