

**Due at 6 pm, Fri. Sep. 30 on BCourses**

Up to 2 people may turn in a single iPython (Jupyter) notebook. Upload .ipynb and answers in pdf.

This exercise is worth 1% grade. (Python exercises will be 9% of grade).

Notes: Use Python 2.7, not 3.

Notebook file is: `www-inst.eecs.berkeley.edu/~ee120/fa16/hwk/PY1-drum.ipynb`

Sound file is: `www-inst.eecs.berkeley.edu/~ee120/fa16/hwk/Tight-High-Tom-1pf.wav`

In this Python exercise, you will try to model the sound of a drum as the unit sample response of a linear difference equation (all-pole model) with infinite impulse response output  $y[n]$ . (Note, the drum is probably non-linear, and the drum stick is not quite an impulse, so results may vary.)

The LDE is given by:

$$\sum_{k=0}^N a_k y[n-k] = b_0 \delta[n] \quad \text{or} \quad y[n] = \frac{1}{a_0} \left( - \sum_{k=1}^N a_k y[n-k] + b_0 \delta[n] \right) \quad (1)$$

Assume initial conditions  $y[n] = 0$  for  $n < 0$ . Let  $a_0 = 1$ .

**1. (10 pts)** Assume  $N = 4$  and given coefficients  $a_1, a_2, a_3, a_4$ . Find  $y[n]$  for  $n = 0, 1, 2, 3$  for unit sample input (note that  $y[0] = b_0/a_0$ ).

**2. (30 pts)** In the `PY1-drum` notebook, add code to the `for` loop to calculate `z` which is  $y[n]$  from eq.(1). Also, update `state[]` which should contain  $[y[n] \ y[n-1] \ \dots \ y[n-N]]^T$ .

Given a sound file, we want to estimate the filter coefficients  $a_k$ . The unknown  $a_k$  can be found from the least square inverse of the system  $V\mathbf{a} = -\mathbf{y}$  where  $V$  are the stacked outputs of the system, and  $\mathbf{y}$  is the vector of outputs. For example, with 6 measurements of  $y[n]$  and 4  $a_k$ , the LDE solution can be expressed as:

$$V\mathbf{a} = \begin{bmatrix} y[0] & 0 & 0 & 0 \\ y[1] & y[0] & 0 & 0 \\ y[2] & y[1] & y[0] & 0 \\ y[3] & y[2] & y[1] & y[0] \\ y[4] & y[3] & y[2] & y[1] \\ y[5] & y[4] & y[3] & y[2] \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} = - \begin{bmatrix} y[1] \\ y[2] \\ y[3] \\ y[4] \\ y[5] \\ y[6] \end{bmatrix} = -\mathbf{y}.$$

The least squares solution for  $\mathbf{a}$  is given by  $\mathbf{a} = -(V^T V)^{-1} V^T \mathbf{y}$

The quality of fit can be quantified by the root-mean-square error:

$$\text{r.m.s. error} = \sqrt{\frac{1}{N} \sum (\hat{y} - y)^2}$$

where  $\hat{y}$  is the LDE output, and  $y$  is the measured data.

**3. (30 pts)** In the `PY1-drum` notebook, choose the size of the state vector `colDim` and number of data samples `rowDim` such that rms error is less than 2500. Note that data is sampled at 44 kHz, so samples are only 22  $\mu\text{s}$  apart.

**4. (20 pts)** Play the `synthDrum.wav` file. Qualitatively describe differences between the sound of the original sampled signal and the reconstructed signal. What might be missing in the reconstructed sound?

**5. (10 pts)** Choose a different percussion sound file from [freewavesamples.com/sample-type/drums/kick](http://freewavesamples.com/sample-type/drums/kick), choose `rowDim`, `colDim` to find a low rms error. Qualitatively describe whether the LDE output is a good match for the original sound. Note sound file source, and show plot comparing  $y, \hat{y}$  over number of data samples.