Professor Fearing EECS120/Problem Set 10 v 1.0 Fall 2016

Due at 4 pm, Fri. Nov. 11 in HW box under stairs (1st floor Cory)

1.**(15 pts) Laplace Transform OW 9.7**

A system with input $x(t)$ and output $y(t)$ is described by the LDE:

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + -3y(t) = x(t) \tag{1}$$

Use the Laplace transform to solve for $y(t)$ with input $x(t) = e^{-t}u(t)$, and initial conditions $y(0-) = 1$ and $\dot{y}(0-) = 2$.

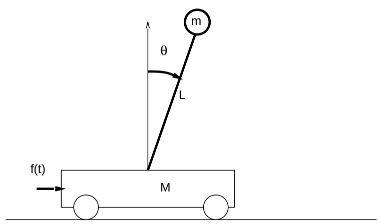

Fig. 1. Broom balancer

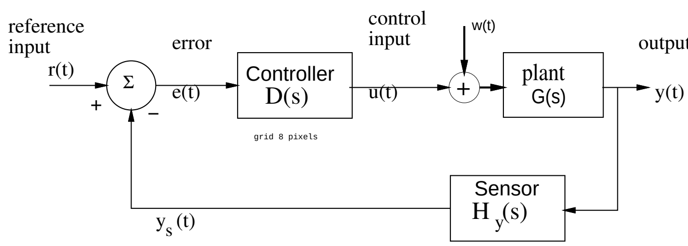

Fig. 2. Control system block diagram

2. (20 pts) (Lec 17, OW 9.5, 9.7)

The differential equation for the broom balancing system (Fig. 1) is given by

$$\frac{1}{3}(4M + m)L\ddot{\theta}(t) = (m + M)g\theta(t) - f(t)$$

under the assumption that $|\theta| << 1$ and the approximation $sin\,\theta = \theta$, and $f(t)$ is the force applied to move the cart.

a. Find the Laplace transform relation for the broom balancing system including both the zero state response and the zero input response.

b. Suppose you can measure $\theta(t)$. You want to balance the broom by choosing $f(t)$ in feedback form as $f(t) = \alpha L\theta(t)$. Will this scheme result in balancing the broom, i.e. so that $\theta(t) \to 0$ as $t \to \infty$, for any small initial condition $\theta(0^-)$ and $\dot{\theta}(0^-)$? Explain why or why not.

c. Suppose you can measure $\theta(t)$ and $\dot{\theta}(t)$. You want to balance the broom by choosing f in feedback form as $f(t) = \alpha L\theta(t) + \beta L\dot{\theta}(t)$ For what values of α and β will this scheme result in balancing the broom, i.e. so that $\theta(t) \to 0$ as $t \to \infty$, for any small initial condition $\theta(0^-)$ and $\dot{\theta}(0^-)$.

3. (15 pts) Feedback Control (Lec 17,18, OW 11-11.2)

Consider the feedback system of Fig.2, with $w(t) = 0$, $D(s) = 1$. Determine the closed loop transfer function $\frac{Y(s)}{R(s)}$, and closed-loop impulse response (i.e. let $r(t) = \delta(t)$) for each of the following system functions in forward and feedback paths:

a) $G(s) = \frac{1}{(s+1)(s+4)}$, $H_y(s) = 1$.

b) $G(s) = \frac{1}{(s+4)}$, $H_y(s) = \frac{1}{s+1}$.

c) $G(s) = \frac{1}{2}$, $H_y(s) = e^{-s/3}$.

4. (25 pts) Feedback Controller Design (Lec 17,18, OW 11-11.2)
Consider the feedback system of Fig.2, with $w(t) = 0$, $H_y(s) = 1$.
a. Suppose $G(s) = \frac{\alpha}{s+\alpha}$ with $\alpha \neq 0$. Show that with proportional control, $D(s) = K$, K can be chosen to stabilize the system, and that $e(t)$ will not tend to zero with $r(t) = u(t)$.
b. Suppose $G(s) = \frac{\alpha}{s+\alpha}$ with $\alpha \neq 0$. Show that with proportional-plus-integral (PI) control, $D(s) = K_1 + \frac{K_2}{s}$, then K_1, K_2 can be chosen to stabilize the system, and that $e(t)$ will tend to zero with $r(t) = u(t)$.
c. Suppose $G(s) = \frac{1}{(s-1)^2}$. Show that with proportional-plus-integral-plus-derivative control, $D(s) = K_1 + \frac{K_2}{s} + K_3 s$, then K_1, K_2, K_3 can be chosen to stabilize the system, and that $e(t)$ will tend to zero with $r(t) = u(t)$. Also show that the system can not be stabilized with a PI controller.

5. (25 pts) Gain and Phase Margin (Lec 18,19, OW 6.5, 9.4, 11.5)
Consider the feedback system of Fig.2, with $w(t) = 0$, $D(s) = 1$. For each part below:
a) Determine the closed loop transfer function $\frac{Y(s)}{R(s)}$.
b) sketch the pole-zero diagram for $G(s)H_y(s)$.
c) Sketch the magnitude and phase Bode plots (e.g. Fig. 11.27) of $G(j\omega)H_y(j\omega)$
d) Roughly estimate the gain and phase margin.

i) $G(s) = \frac{10s+1}{s^2+s+1}$, $H_y(s) = 1$.

ii) $G(s) = \frac{s/10+1}{s^2+s+1}$, $H_y(s) = 1$.

iii) $G(s) = \frac{1}{(s+3)^3}$, $H_y(s) = \frac{1}{s+3}$.

iv) $G(s) = \frac{1}{(s+1)^2(s+10)}$, $H_y(s) = 100$.