Due at 4 pm, Fri. Dec. 2, 2016 in HW box under stairs (1st floor Cory)

1. (20 pts) Digital LPF design (OW 10.4, 10.9, Lec 20)

Design a causal low pass filter $H(z)$, such that $|H(e^{j0})| = 1$, $|H(e^{j\pi/4})| = 0$ and $|H(e^{j\pi})| = 0$. Sketch pole-zero plot and use geometric approximations to sketch $|H(e^{j\Omega})|$ and $\angle H(e^{j\Omega})$. Find the difference equation corresponding to this $H(z)$ and show that it is causal.

2. (20 pts) All-pass filter. (Lec. 22, `minphase.pdf` on Piazza)

A discrete time causal LTI system has transfer function:

$$H(z) = \frac{(1 + 0.4z^{-1})(1 + 2z^{-1})}{(1 - 0.9z^{-1})(1 - 0.8z^{-1})}$$

a) Draw the pole-zero diagram for $H(z)$. Is the system stable?

b) Find the minimum phase system $H_{min}(z)$ and an all-pass system $H_{ap}(z)$ such that $H(z) = H_{min}(z)H_{ap}(z)$ and plot the respective pole-zero diagrams.

c) Find a realizable, causal inverse filter $H_{inv}(z)$ such that $|H_{inv}(e^{j\Omega})||H(e^{j\Omega})| = 1$ for all Ω.

3. (20 pts) Z transform application (Ch 10)

Jane gets a loan with principal balance at start of year n is $p[n]$. Starting balance $p[0] = \$500,000$. The loan is interest only with an annual interest rate of 5%. The loan is negative amortization in year n if the annual payment is less than $0.05p[n]$. If the annual payment is greater than $0.05p[n]$, the principal balance is reduced. At year $n = 0$, Jane has an annual salary of $s[0] = \$100,000$. Jane's salary increases by 3% per year, and she uses 15% of her gross salary to pay interest and principal on the loan.

a) Solve for $p[n]$ using Z-transforms. When does the loan get paid off? What is the total paid?

b) What fixed per cent of her gross salary must Jane use to pay off the loan in 30 years?

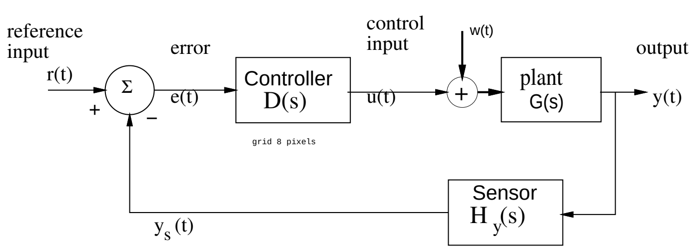

Fig 1

4. (20 pts) Steady State Error (Lec 23, Steady State Error handout)

For the system in Fig 1, let $D(s) = 1$, $G(s) = \frac{500(s+0.5)}{s^2(s+10)^2}$, and $H_y(s) = 1$. Assume $w(t) = 0$.

a) What is the system type? (type 0,1,2,?)

b) What input waveform $r(t)$ would yield a constant error? (e.g. step, ramp, parabola, or ?)

c) Assuming stability, what is the steady state error for a unit input of the type of $r(t)$ found in b)?

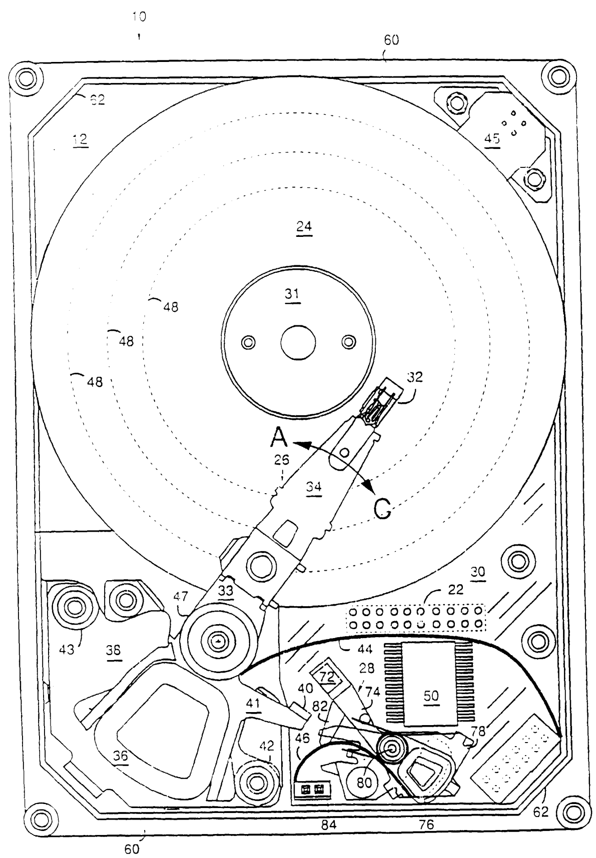

Fig. 2

5. (20 pts) Internal Model Principle (Lec 23, `InternalModPrinc.pdf` on Piazza)
A voice coil assembly for a disk drive is shown in Fig. 2. The electromagnet **36** rotates the read/write head arm **34** about rotary bearing **47**. The head position $y(t)$ is controlled on the arc A-C to select tracks. Consider the system of Fig. 1, where the plant $G(s)$ is the transfer function from force to position, described by the LDE $f(t) = m\ddot{y}(t)+b\dot{y}(t)$ where $f(t) = u(t)+w(t)$ is the force from the voice coil and the disturbance, m is the mass of the head assembly, and b is damping. Assume $m = 1$ and $b = 10$. Assume the head position is read directly with $H_y(s) = 1$, and that the desired track position is given by $r(t)$. (Recent disk drives have a track spacing less than 100 nm.)
Consider Proportional+Integral Control, with $D(s) = \frac{k_p s+k_I}{s}$, with $k_p = 30$ and $k_I = 30$.
a) Show that a step input $r(t)$ can be tracked with zero asymptotic error, (i.e. $e(t) \to 0$ as $t \to \infty$).
b) As the disk rotates on its bearing, wobble is possible. This can be modelled as a disturbance $w(t) = 0.01\sin(4t)$ for $t > 0$. Find the peak-to-peak error in position due to this disturbance.
c) A new controller is proposed which includes a "model" of the disturbance. Consider a new

$$D(s) = \frac{k_p s + k_I}{s}\frac{100(s^2 + 2s + 17)}{s^2 + 16}.$$

Find $\frac{E(s)}{W(s)}$ and find the peak-to-peak error in position due to the $w(t)$ this disturbance.