# EE120 Fall 2016

HOMEWORK 3 Solutions

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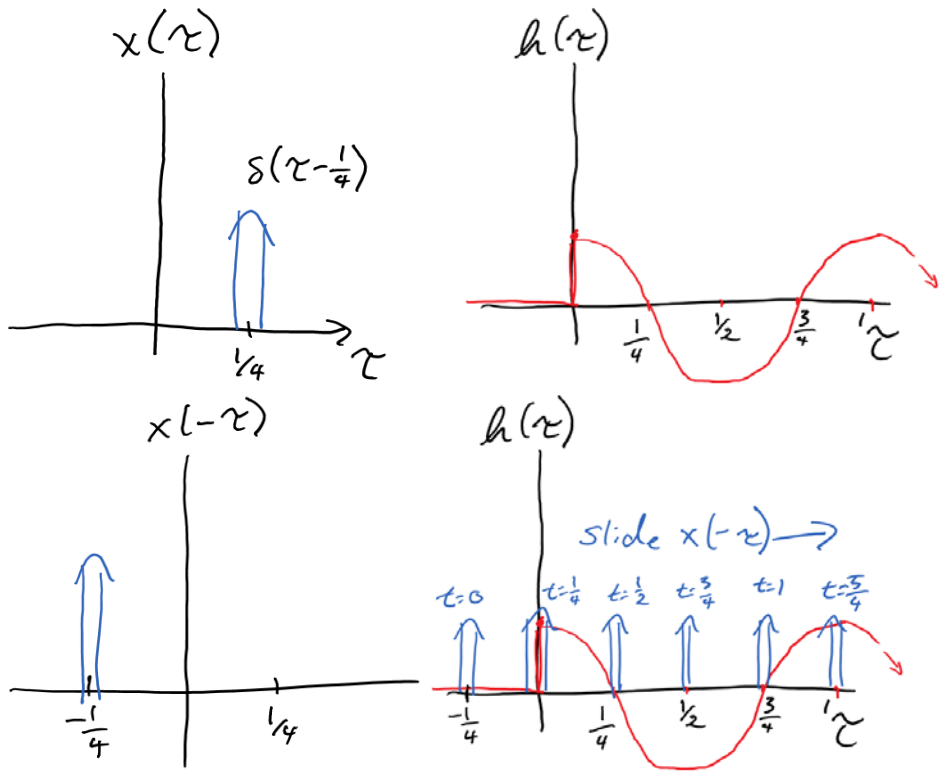
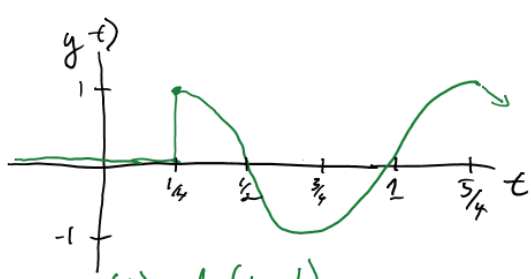
## Problem 1 Solution

For each solution, draw $x(\tau)$ and flip around $\tau = 0$, then slide the result across $h(\tau)$.

### 1.a.



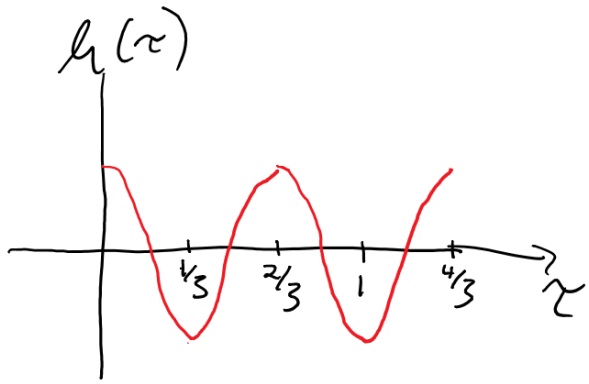



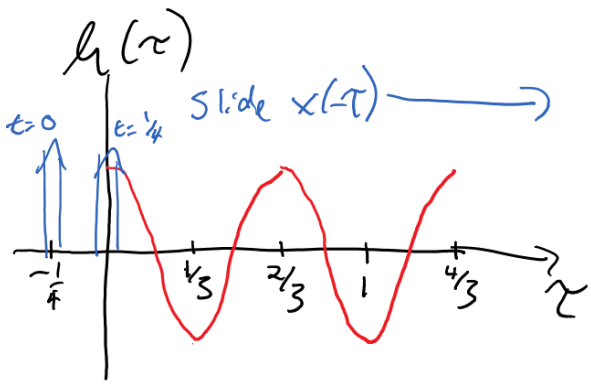

$$y(t) = h\left(t - \frac{1}{4}\right)$$

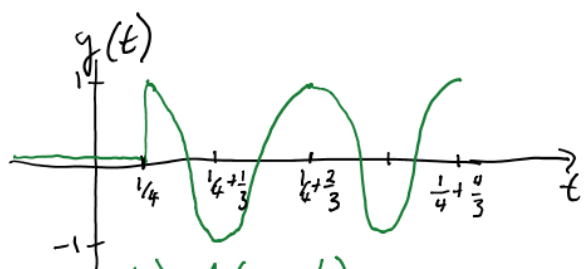
$$y(t) = u\left(t - \frac{1}{4}\right)\cos\left(2\pi t - \frac{\pi}{2}\right)$$

## 1.b

Use the same x as part a.



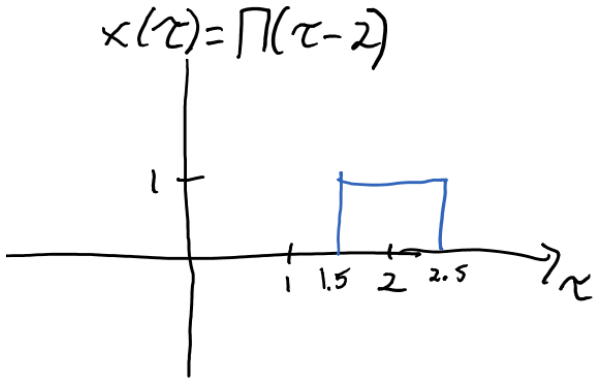



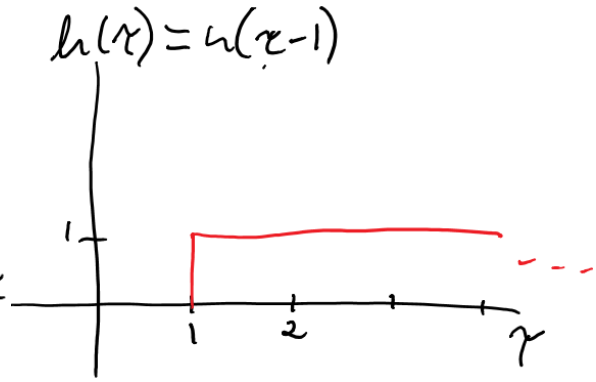



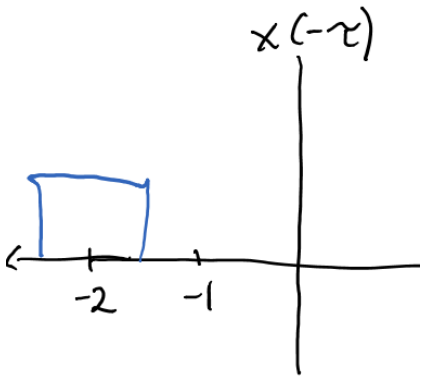

$y(t) = h(t - \frac{1}{4})$

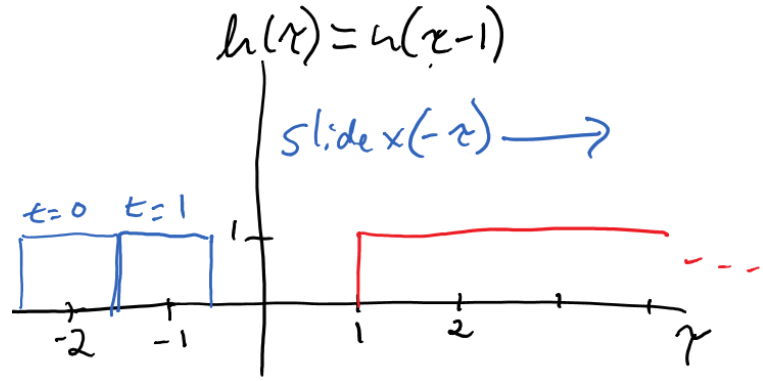
$y(t) = u(t - \frac{1}{4}) \cos(3\pi t - \frac{3\pi}{4})$

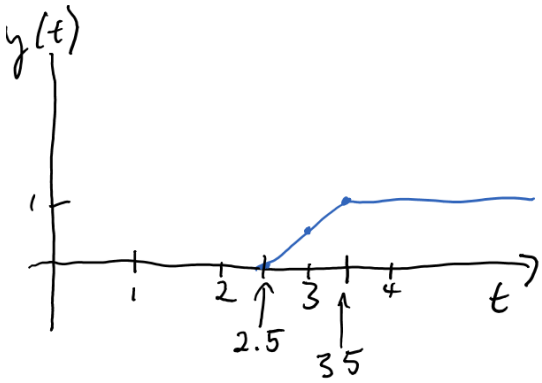
1.c


x(τ) = Π(τ − 2)
1
1
1.5
2
2.5
τ

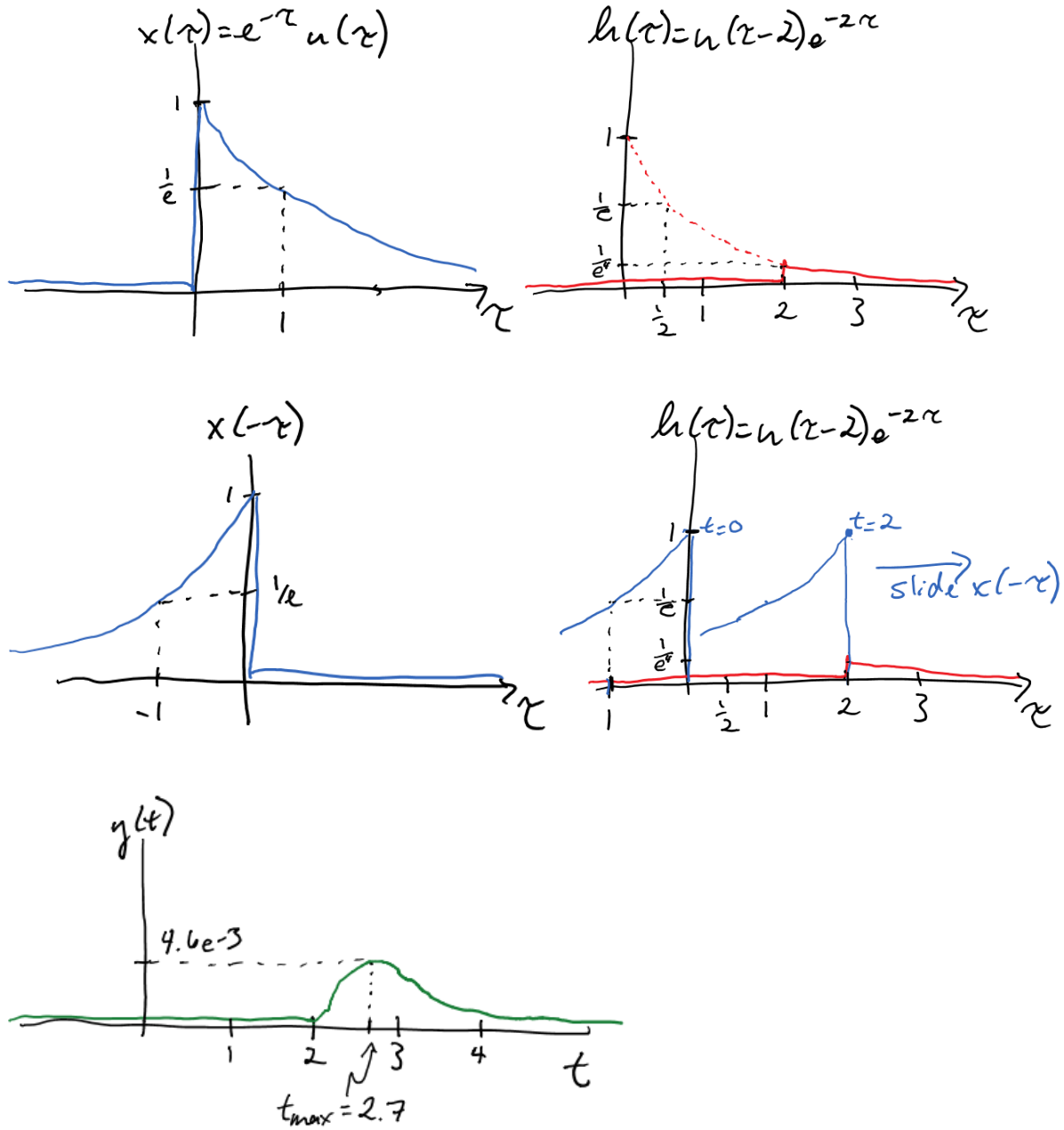

h(τ) = u(τ − 1)
1
1
2
τ


x(−τ)
−2
−1


h(τ) = u(τ − 1)
slide x(−τ)
t = 0
t = 1
1
−2
−1
1
2
τ


y(t)
1
1
2
3
4
t
2.5
3.5

## 1.d




Analytical convolution gives:

$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau = \int_{-\infty}^{\infty} \exp\big(-(t-\tau)\big)\, u(t-\tau)\exp(-2\tau)\, u(\tau-2)d\tau$

The integrand is zero unless $t-\tau > 0$; and $\tau - 2 > 0$; which require the integration variable, $\tau$, to be $\tau < t$ and $\tau > 2$.

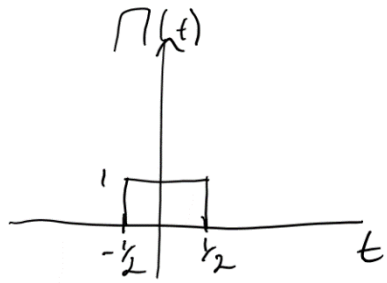
$y(t) = \left[\int_2^t \exp\big(-(t-\tau)\big)\exp(-2\tau)\, d\tau\right] \cdot u(t-2) = \left[\exp(-t)\int_2^t \exp(-\tau)\, d\tau\right] \cdot u(t-2)$

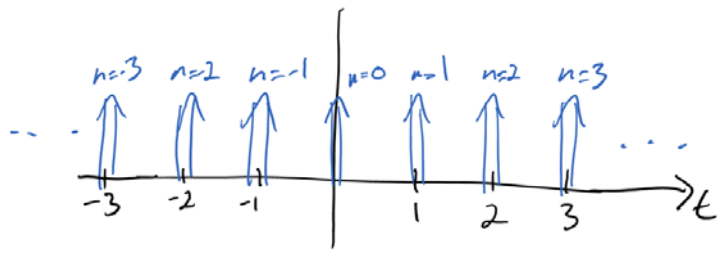
$y(t) = [\exp(-2-t) - \exp(-2t)] \cdot u(t-2)$

This gives the sketch above. We include the u(t-2) to account for the zero overlap of x and h before t = 2.

## Problem 2 Solution

$\Pi(t) = u\left(t + \frac{1}{2}\right) - u\left(t - \frac{1}{2}\right)$ $\qquad$ $comb(t) = \sum_{n=-\infty}^{\infty} \delta(t - n)$



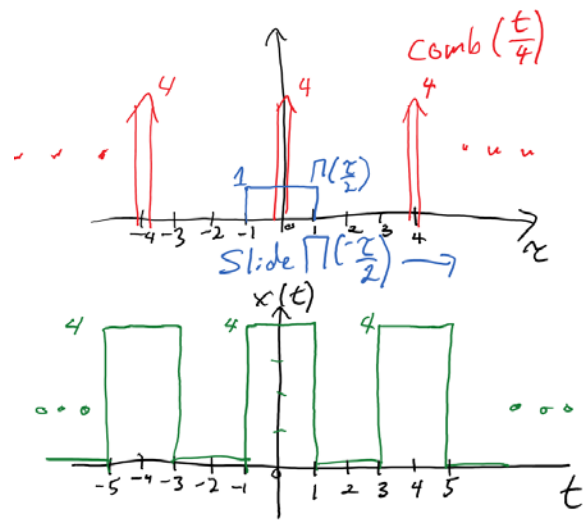

### Part a)

$\Pi\left(\frac{t}{2}\right) = u\left(\frac{t}{2} + \frac{1}{2}\right) - u\left(\frac{t}{2} - \frac{1}{2}\right)$: This is a box of height 1 with width 2, centered at zero

$comb\left(\frac{t}{4}\right) = \sum_{n=-\infty}^{\infty} \delta(\frac{1}{4}(t - 4n)) = 4\sum_{n=-\infty}^{\infty} \delta(t - 4n)$: This is a delta train with spacing 4, and delta height = 4.

$$x_1(\mathrm{t}) = \Pi\left(\frac{t}{2}\right) * comb\left(\frac{t}{4}\right) = \int_{-\infty}^{\infty} \Pi\left(\frac{\tau}{2}\right) comb\left(\frac{t-\tau}{4}\right) d\tau$$

Using "flip and drag" method for convolution, $x_1(\mathrm{t})$ will be a periodic function with a repeating box of width 2, height 4, fundamental period $T_0 = 4$, and fundamental frequency $\omega_0 = \frac{\pi}{2}$:




To find $a_k$, we use the analysis equation.

$$a_k = \frac{1}{4}\int_{-2}^{2} x_1(t) \exp\left(-jk\frac{\pi}{2}t\right) dt$$

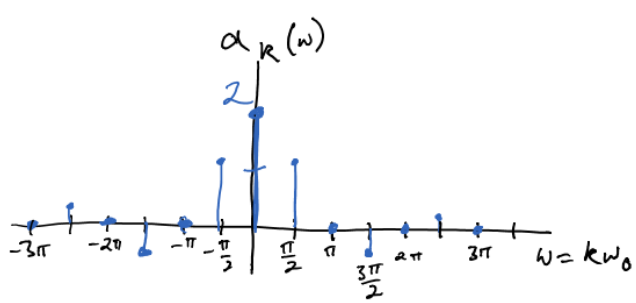
$$a_k = \frac{1}{4}\int_{-1}^{1} 4 \exp\left(-jk\frac{\pi}{2}t\right) dt$$

$$a_k = \int_{-1}^{1} \exp\left(-jk\frac{\pi}{2}t\right) dt = \frac{4}{k\pi}\frac{\left[\exp\left(jk\frac{\pi}{2}\right) - \exp\left(-jk\frac{\pi}{2}\right)\right]}{2j}$$

$$\boxed{a_k = \frac{4}{\pi k}\sin\left(\frac{k\pi}{2}\right)}$$

For $k = 0$, we solve the integral: $a_0 = \frac{1}{4}\int_{-1}^{1} 4dt$

$$\boxed{a_0 = \int_{-1}^{1} 1dt = 2}$$



## Part b)

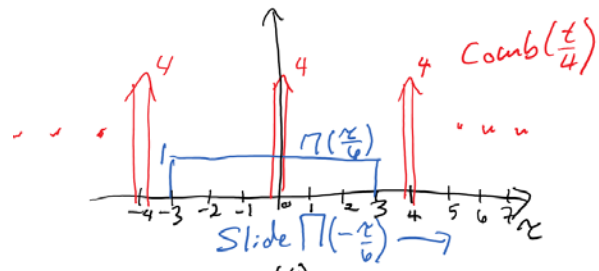
$\Pi\left(\frac{t}{6}\right) = u\left(\frac{t}{6}+\frac{1}{2}\right) - u\left(\frac{t}{6}-\frac{1}{2}\right)$: This is a box of height 1 with width 6, centered at zero

$comb\left(\frac{t}{4}\right) = \sum_{n=-\infty}^{\infty} \delta(\frac{1}{4}(t-4n)) = 4\sum_{n=-\infty}^{\infty} \delta(t-4n)$: This is a delta train with spacing 4, and delta height = 4.

$$x_2(\mathrm{t}) = \Pi\left(\frac{t}{6}\right) * comb\left(\frac{t}{4}\right) = \int_{-\infty}^{\infty} \Pi\left(\frac{\tau}{6}\right) comb\left(\frac{t-\tau}{4}\right) d\tau$$

Using "flip and drag" method for convolution, $x_1(\mathrm{t})$ will be a square wave with a low level of 4 and high level of 8, fundamental period $T_0 = 4$, and fundamental frequency $\omega_0 = \frac{\pi}{2}$:

From this flip-and-slide result, we can see that the function $x_2(t)$ can be written: $x_2(t) = 8 - 4\Pi\left(\frac{t}{2}\right)$



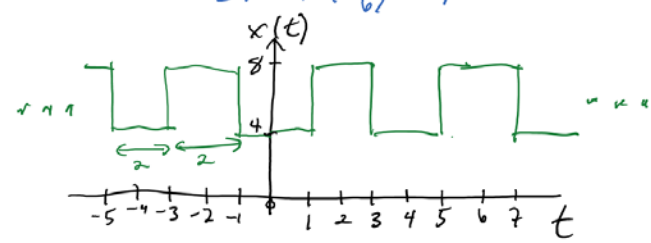

To find $a_k$, we use the analysis equation. $a_k = \frac{1}{4}\int_{-2}^{2} x_2(t)\exp\left(-jk\frac{\pi}{2}t\right)dt$

$$a_k = \frac{1}{4}\int_{-2}^{2}\left[8 - 4\Pi\left(\frac{t}{2}\right)\right]\exp\left(-jk\frac{\pi}{2}t\right)dt$$

$$a_k = 2\int_{-2}^{2}\exp\left(-jk\frac{\pi}{2}t\right)dt - \int_{-2}^{2}\Pi\left(\frac{t}{2}\right)\exp\left(-jk\frac{\pi}{2}t\right)dt$$

$$a_k = \frac{4\sin(k\pi)}{k\frac{\pi}{2}} - \frac{4\sin\left(\frac{k\pi}{2}\right)}{k\pi}$$

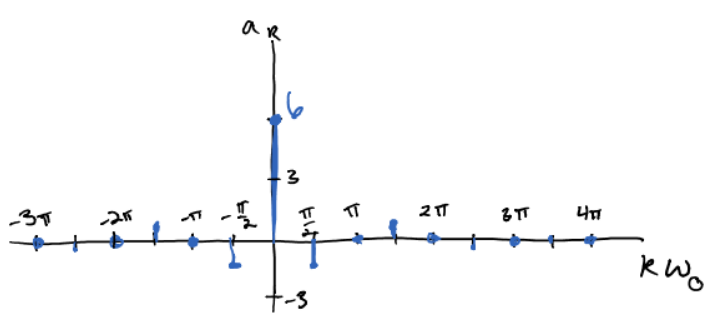
$$\boxed{a_k = -\frac{4\sin\left(\frac{k\pi}{2}\right)}{k\pi}\text{; for } k \neq 0.}$$




For $k = 0$, we can solve the integral:

$$a_0 = \frac{1}{4}\int_{-2}^{2}\left[8 - 4\Pi\left(\frac{t}{2}\right)\right]dt = \int_{-2}^{2}\left[2 - \Pi\left(\frac{t}{2}\right)\right]dt = 8 - \int_{-2}^{2}\left[\Pi\left(\frac{t}{2}\right)\right]dt = 8 - \int_{-1}^{1} 1dt = 6$$

$$\boxed{a_0 = 6}$$




Using L'Hopital's rule on the final expression for $a_k = -\frac{4\sin\left(\frac{k\pi}{2}\right)}{k\pi}$ does not give the correct answer. We have to use the previous expression to use L'Hopital properly: $a_k = \frac{4\sin(k\pi)}{k\frac{\pi}{2}} - \frac{4\sin\left(\frac{k\pi}{2}\right)}{k\pi}$. This result should also give $a_0 = 6$. **Even though the boxed answer is correct for non-zero k, it is evident that evaluating at k = 0 will give: 0/0 – 0/0. This should be a cue that we will need to apply L'Hopital's rule on both terms!!**

## Part c)

$x_3(\mathrm{t}) = \Pi\left(\frac{t}{2}\right) * \Pi(t-1) * comb\left(\frac{t}{4}\right) = \Pi(t-1) * x_1(t)$

$x_1(t)$ is the same as in part a. $\Pi(t-1)$ is a box function of width = 1, height = 1, centered at t = 1.

We know from Part a) that the Fourier coefficients of $x_1(t)$ are:$a_k = \frac{4}{\pi k}\sin\left(\frac{k\pi}{2}\right)$

If we treat $\Pi(t-1)$ as an impulse response of an LTI system, $h(t) = \Pi(t-1)$, the output of this system for an arbitrary input $x(\mathrm{t})$ can be written as:

$y(\mathrm{t}) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)dt$

For $x(t) = \exp(j\omega t)$, $y(\mathrm{t}) = \int_{-\infty}^{\infty} exp\big(j\omega(t-\tau)\big)\,\Pi(\tau-1)d\tau$

$y(\mathrm{t}) = \exp(j\omega t)\int_{1/2}^{3/2} exp(-j\omega\tau)\,dt = \frac{2\exp(j\omega t)}{-2j\omega}\left[exp\left(-j\omega\frac{3}{2}\right) - \ exp\left(-j\omega\frac{1}{2}\right)\right]$

$y(\mathrm{t}) = \frac{\sin(\omega/2)}{\omega/2}\frac{1}{\exp(j\omega)}\exp(j\omega t)$

So, $\{\exp(j\omega t)\} = \frac{\sin(\omega/2)}{\omega/2}\frac{1}{\exp(j\omega)}$. Now, we use our synthesis equation for $x_1(t)$:

$x_1(t) = \sum_{k=-\infty}^{\infty} a_k \exp\left(jk\frac{\pi}{2}t\right)$

Write output y using $x_1(t)$

$y(\mathrm{t}) = H\{x_1(t)\} = H\left\{\sum_{k=-\infty}^{\infty} a_k \exp\left(jk\frac{\pi}{2}t\right)\right\} = \sum_{k=-\infty}^{\infty} a_k H\left\{\exp\left(jk\frac{\pi}{2}t\right)\right\}$

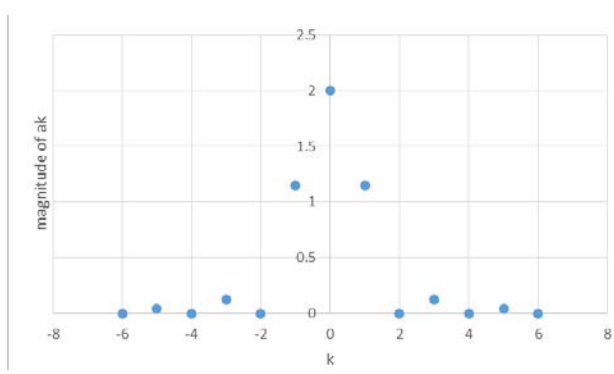
Plug in $k\frac{\pi}{2}$ for $\omega$ in the transfer function: $H\left\{\exp\left(jk\frac{\pi}{2}t\right)\right\} = \frac{\sin\left(k\frac{\pi}{2}/2\right)}{k\frac{\pi}{2}/2}\frac{1}{\exp\left(jk\frac{\pi}{2}\right)}\exp\left(jk\frac{\pi}{2}t\right)$

$y(\mathrm{t}) = \sum_{k=-\infty}^{\infty} a_k \frac{\sin\left(k\frac{\pi}{2}/2\right)}{k\frac{\pi}{2}/2}\frac{1}{\exp\left(jk\frac{\pi}{2}\right)}\exp\left(jk\frac{\pi}{2}t\right) = \sum_{k=-\infty}^{\infty} a'_k \exp\left(jk\frac{\pi}{2}t\right)$

So, the new $\boxed{a'_k = a_k \frac{\sin\left(k\frac{\pi}{2}/2\right)}{k\frac{\pi}{2}/2}\frac{1}{\exp\left(jk\frac{\pi}{2}\right)} = \frac{16\sin\left(k\frac{\pi}{2}\right)\sin\left(k\frac{\pi}{4}\right)}{\pi^2 k^2}\exp\left(-jk\frac{\pi}{2}\right)}$

$a'_0 = 2$

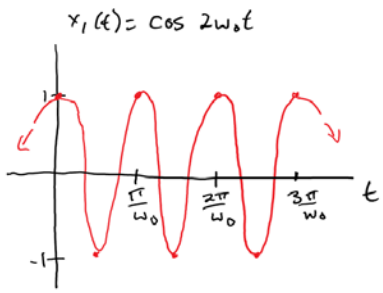
$a'_{even} = 0$

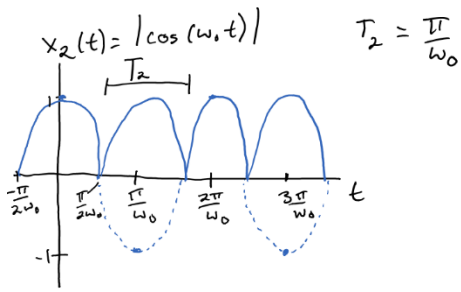


# Problem 3 Solution

## Part A: Fourier coefficients

$x_1(t) = \cos(2\omega_0 t)$ $\qquad$ $x_2(t) = |\cos(\omega_0 t)|$




*Find Fourier coefficients of $x_1$:*

$x_1(t)$ is periodic with fundamental period, $T = \frac{2\pi}{2\omega_0} = \frac{\pi}{\omega_0}$

$$a_k = \frac{\omega_0}{\pi}\int_0^{\frac{\pi}{\omega_0}} \cos(2\omega_0 t)\exp(-jk2\omega_0 t)\,dt$$

$$a_k = \frac{\omega_0}{\pi}\int_0^{\frac{\pi}{\omega_0}} \frac{\exp(j2\omega_0 t)+\exp(-j2\omega_0 t)}{2}\exp(-jk2\omega_0 t)\,dt$$

$$a_k = \frac{\omega_0}{2\pi}\int_0^{\frac{\pi}{\omega_0}} [\exp(j\omega_0 2(1-k)t) + \exp(-j\omega_0 2(1+k)t)]dt$$

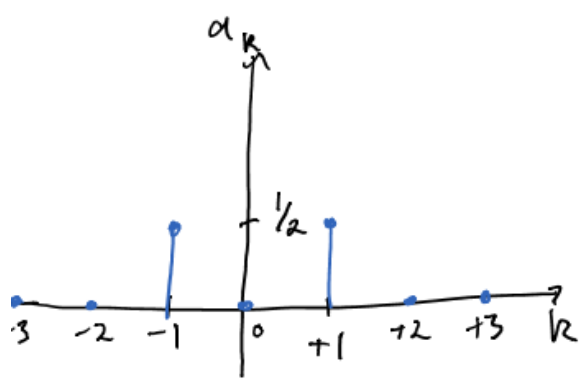
Solve for k = +1 and -1,

$$a_1 = \frac{\omega_0}{2\pi}\int_0^{\frac{\pi}{\omega_0}} [1 + \exp(-j\omega_0 4t)]dt$$

$$a_1 = \frac{\omega_0}{2\pi}\left[t + \frac{\exp(-j\omega_0 4t)}{-j\omega_0 4}\right]\Big|_0^{\frac{\pi}{\omega_0}}$$

$$\boxed{a_1 = \frac{\omega_0}{2\pi}\left[\frac{\pi}{\omega_0}\right] = \frac{1}{2}}$$

Substituting k = -1 leads to the same result, $a_{-1} = \frac{1}{2}$




Solve for other k,

$$a_k = \frac{\omega_0}{2\pi}\left[\frac{\exp(j\omega_0 2(1-k)t)}{j\omega_0 2(1-k)} + \frac{\exp(-j\omega_0 2(1+k)t)}{-j\omega_0 2(1+k)}\right]\Big|_0^{\frac{\pi}{\omega_0}}$$

$$a_k = \frac{1}{4\pi j}\left[\frac{\exp(j(1-k)2\pi)-1}{(1-k)} + \frac{\exp(-j(1+k)2\pi)-1}{-(1+k)}\right]$$

$$\boxed{a_k = 0 \text{ for } k \neq \pm 1.}$$

Since the phase of the complex exponentials is always a positive or negative integer multiple of $2\pi$, the sums always evaluate to 0 for all k except k = +1 or -1.

*Next, find Fourier coefficients for $x_2$:*

$x_2(t)$ is periodic with fundamental period, $T_2 = \frac{\pi}{\omega_0}$

$$a_k = \frac{\omega_0}{\pi}\int_{-\frac{\pi}{2\omega_0}}^{\frac{\pi}{2\omega_0}} \cos(\omega_0 t)\exp(-jk2\omega_0 t)\,dt$$

$$a_k = \frac{\omega_0}{\pi}\int_{-\frac{\pi}{2\omega_0}}^{\frac{\pi}{2\omega_0}} \frac{\exp(j\omega_0 t)+\exp(-j\omega_0 t)}{2}\exp(-jk2\omega_0 t)\,dt$$

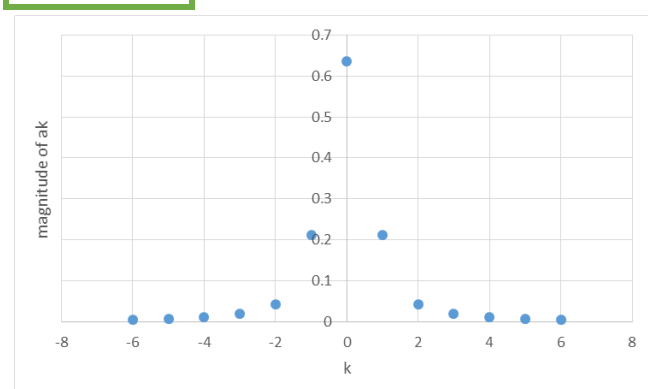
$$a_k = \frac{\omega_0}{2\pi}\int_{-\frac{\pi}{2\omega_0}}^{\frac{\pi}{2\omega_0}} [\exp(j\omega_0(1-2k)t) + \exp(-j\omega_0(1+2k)t)]dt$$

$$a_k = \frac{\omega_0}{2\pi}\left(\frac{\exp(j\omega_0(1-2k)t)}{j\omega_0(1-2k)} + \frac{\exp(-j\omega_0(1+2k)t)}{-j\omega_0(1+2k)}\right)\Big|_{-\frac{\pi}{2\omega_0}}^{\frac{\pi}{2\omega_0}}$$

$$a_k = \frac{1}{2\pi}\frac{1}{j}\left(\frac{\exp\left(j(1-2k)\frac{\pi}{2}\right)-\exp\left(-j(1-2k)\frac{\pi}{2}\right)}{(1-2k)} + \frac{\exp\left(-j(1+2k)\frac{\pi}{2}\right)-\exp\left(j(1+2k)\frac{\pi}{2}\right)}{-(1+2k)}\right)$$

$$a_k = \frac{1}{\pi}\left(\frac{\sin\left((1-2k)\frac{\pi}{2}\right)}{(1-2k)} + \frac{\sin\left((1+2k)\frac{\pi}{2}\right)}{(1+2k)}\right)$$

$$\boxed{\begin{aligned} a_k &= \frac{2}{\pi}\frac{(-1)^k}{1-4k^2} \\ a_0 &= \frac{2}{\pi} \end{aligned}}$$



## Part B: Time averaged power

We will use $P_{avg,x} = \frac{1}{T}\int_{<T>}|x(t)|^2 dt$

*Find time averaged power for $x_1(t)$*

$x_1(t) = \cos(2\omega_0 t)\,; T = \frac{\pi}{\omega_0}$

$$P_{avg,x1} = \frac{1}{T}\int_{<T>}|x_1(t)|^2 dt$$

$$P_{avg,x1} = \frac{\omega_0}{\pi}\int_0^{\frac{\pi}{\omega_0}}|\cos(2\omega_0 t)|^2 dt = \frac{\omega_0}{\pi}\int_0^{\frac{\pi}{\omega_0}}\cos^2(2\omega_0 t)\,dt$$

$$P_{avg,x1} = \frac{\omega_0}{\pi}\int_0^{\frac{\pi}{\omega_0}}\frac{1+\cos(2\cdot 2\omega_0 t)}{2}\,dt$$

$$P_{avg,x1} = \frac{\omega_0}{\pi}\left[\frac{t}{2} + \frac{\sin(4\omega_0 t)}{8\omega_0}\right]\Big|_0^{\frac{\pi}{\omega_0}}$$

$$\boxed{P_{avg,x1} = \frac{\omega_0}{\pi}\left[\frac{\pi}{2\omega_0}\right] = \frac{1}{2}}$$

*Find time averaged power for $x_2(t)$*

$x_2(t) = |\cos(\omega_0 t)|; T = \frac{\pi}{\omega_0}$

$$P_{avg,x2} = \frac{1}{T}\int_{<T>}|x_2(t)|^2 dt$$

$$P_{avg,x2} = \frac{\omega_0}{\pi}\int_0^{\frac{\pi}{\omega_0}}|\cos(\omega_0 t)|^2 dt = \frac{\omega_0}{\pi}\int_0^{\frac{\pi}{\omega_0}}\cos^2(\omega_0 t)\,dt$$

$$P_{avg,x2} = \frac{\omega_0}{\pi}\int_0^{\frac{\pi}{\omega_0}}\frac{1+\cos(2\cdot\omega_0 t)}{2}\,dt$$

$$P_{avg,x2} = \frac{\omega_0}{\pi}\left[\frac{t}{2} + \frac{\sin(2\omega_0 t)}{4\omega_0}\right]\Big|_0^{\frac{\pi}{\omega_0}}$$

$$\boxed{P_{avg,x2} = \frac{\omega_0}{\pi}\left[\frac{\pi}{2\omega_0}\right] = \frac{1}{2}}$$

## Part C: Power at Desired Frequency

The desired frequency is $2\omega_0$. The k[th] Fourier coefficient for $x_2$ corresponds to a frequency of $\frac{2\pi k}{T_2}$. $T_2$ is the fundamental period of $x_2$, $T_2 = \frac{\pi}{\omega_0}$.

$$\frac{2\pi k}{T_2} = 2\omega_0$$

$$k = \frac{T_2\omega_0}{\pi} = 1$$

Sum of square of Fourier coefficients k = +1 and k = -1:

$$a_1 = \frac{1}{\pi}\left(1 - \frac{1}{3}\right) = \frac{2}{3\pi}$$

$$a_{-1} = \frac{1}{\pi}\left(\frac{-1}{3} + 1\right) = \frac{2}{3\pi}$$

$$|a_1|^2 + |a_{-1}|^2 = \frac{8}{9\pi^2} \cong 0.09$$

By Parseval's Theorem for continuous time periodic signals (OW section 3.5.7, p.205), we know the sum of squares of the fourier coefficients must be equal to the average power in the periodic signal. The average power is shown in Part b:

$$P_{avg,x2} = \frac{1}{2}$$

Neglecting $a_0 = \frac{2}{\pi}$ means subtracting $|a_0|^2$ from $P_{avg,x2}$

$$P_{avg,x2} - |a_0|^2 = \frac{1}{2} - \frac{4}{\pi^2} \cong 0.0947$$

Therefore, the fraction of time averaged power at $2\omega_0$ (neglecting the DC term) is approximately

$$\boxed{\frac{|a_1|^2+|a_{-1}|^2}{0.0947} = \frac{0.09}{0.0947} = 0.95 = 95\%}$$

## Part D: LTI Filter with $x_2(t)$

If we have a low pass filter, $H(j\omega) = \frac{1}{1+j\frac{\omega}{2\omega_0}}$

And we know that $x_2(t)$ is expressed by its Fourier coefficients in the form,

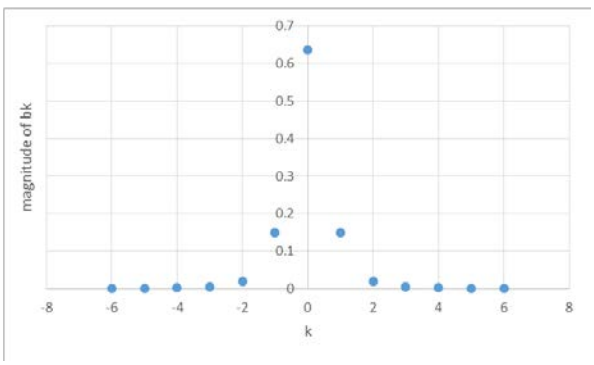
$$x_2(t) = \sum_{k=-\infty}^{\infty} a_k \exp(jk2\omega_0 t)$$

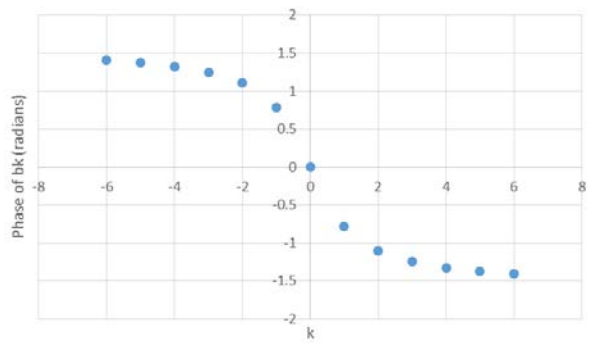
Then the output of the system, $y(t)$, is given by

$$y(t) = \sum_{k=-\infty}^{\infty} a_k H(jk2\omega_0) \exp(jk2\omega_0 t)$$

The Fourier series coefficients of $y(t)$ are $b_k$:

$$b_k = a_k H(jk2\omega_0) = a_k \frac{1}{1+jk} = \frac{2}{\pi}\frac{(-1)^k}{1-4k^2}\frac{1}{1+jk}$$




$$|b_k|^2 = \frac{4}{(1+k^2)\pi^2}\frac{1}{(1-4k^2)^2}$$

The DC term is $b_0 = \frac{2}{\pi}$. The Fourier coefficients of $y(t)$ for k = +1 and -1 are:

$$b_1 = \frac{2}{3\pi}\frac{1}{1+j} \qquad |b_1|^2 = \frac{2}{9\pi^2}$$

$$b_{-1} = \frac{2}{3\pi}\frac{1}{1-j} \qquad |b_{-1}|^2 = \frac{2}{9\pi^2}$$

The fraction of power in the k = +1 and k = -1 components is given (where the total power minus DC is given by summing all $b_k$'s by computer calculation, no simple closed form solution):

$$\boxed{\frac{|b_1|^2+|b_{-1}|^2}{0.046} \cong 0.98 = 98\%}$$

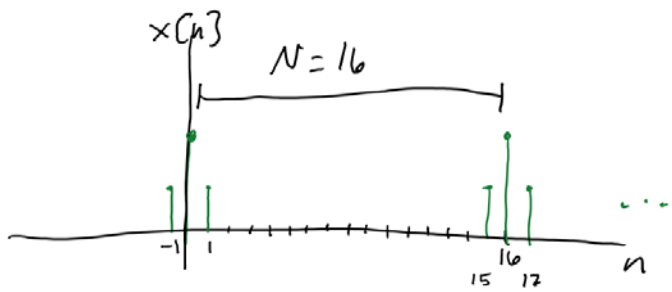
Truncating the series at k = +6 and -6 gives a good estimate, since the magnitude of $b_k$ is proportional to $1/k^4$.

By using a filter, we get a signal where more of the total power is concentrated at the desired frequency.

# Problem 4 Solution

## Part A:

$x[n] = \frac{1}{2}\delta[n+1] + \delta[n] + \frac{1}{2}\delta[n-1]$, N = 16




Using the analysis equation for discrete time signals:

$$a_k \triangleq \frac{1}{N}\sum_{<N>} x[n]\exp\left(-jk\frac{2\pi}{N}n\right)$$

Select a period from n = -7 to n = +8

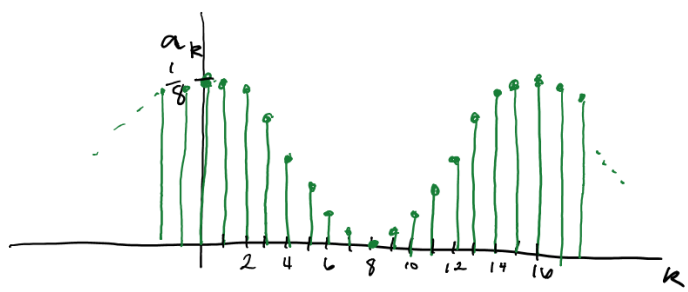
$$a_k = \frac{1}{16}\sum_{n=-7}^{8}\left(\frac{1}{2}\delta[n+1] + \delta[n] + \frac{1}{2}\delta[n-1]\right)\exp\left(-jk\frac{2\pi}{16}n\right)$$

Since x is non-zero only when n = -1, 0, or +1, the sum can be written:

$$a_k = \frac{1}{16}\left[\frac{1}{2}\exp\left(-jk\frac{2\pi}{16}(-1)\right) + \exp\left(-jk\frac{2\pi}{16}(0)\right) + \frac{1}{2}\exp\left(-jk\frac{2\pi}{16}(1)\right)\right]$$

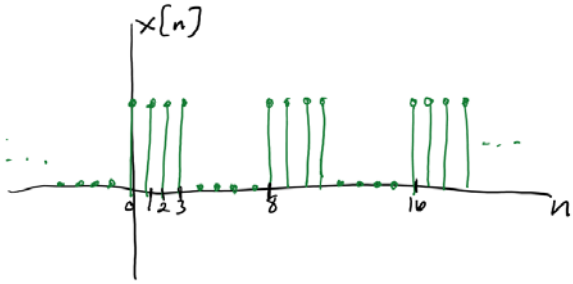
$$a_k = \frac{1}{16}\left[1 + \frac{1}{2}\left(\exp\left(jk\frac{2\pi}{16}\right) + \exp\left(-jk\frac{2\pi}{16}\right)\right)\right]$$

$$\boxed{a_k = \frac{1}{16}\left[1 + \cos\left(2\pi\frac{k}{16}\right)\right]}$$



## Part B:

$x[n] = [1,1,1,1,0,0,0,0]$, N = 8




Using the analysis equation for discrete time signals:

$$a_k \triangleq \frac{1}{N}\sum_{<N>} x[n]\exp\left(-jk\frac{2\pi}{N}n\right)$$

Select a period from n = 0 to n = +7

$$a_k = \frac{1}{8}\sum_{n=0}^{7}(\delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3])\exp\left(-jk\frac{2\pi}{8}n\right)$$

$$a_k = \frac{1}{8}\sum_{n=0}^{3}\exp\left(-jk\frac{\pi}{4}n\right) = \frac{1}{8}\sum_{n=0}^{3}W^n = \frac{1}{8}\left(\frac{1-W^4}{1-W}\right)$$

$$a_k = \frac{1}{8}\left(\frac{1-\exp(-jk\pi)}{1-\exp\left(-\frac{jk\pi}{4}\right)}\right) = \frac{1}{8}\exp\left(jk\frac{5\pi}{8}\right)\frac{\sin\left(\frac{k\pi}{2}\right)}{\sin\left(\frac{k\pi}{8}\right)}$$

$$\boxed{|a_k| = \frac{1}{8}\left|\frac{\sin\left(\frac{k\pi}{2}\right)}{\sin\left(\frac{k\pi}{8}\right)}\right|}$$

Since x is non-zero only when n = 0, 1, 2, or 3, the sum could also be written:

$$a_k = \frac{1}{8}\left[\exp\left(-jk\frac{2\pi}{8}(0)\right) + \exp\left(-jk\frac{2\pi}{8}(1)\right) + \exp\left(-jk\frac{2\pi}{8}(2)\right) + \exp\left(-jk\frac{2\pi}{8}(3)\right)\right]$$

For k = 0, the sum simplifies:

$$a_0 = \frac{1}{8}[1+1+1+1] = \frac{1}{2}$$