Due at 4 pm, Fri. Sep. 23 in HW box under stairs (1st floor Cory)
Reading: O&W Ch3, Ch4.

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1. (22 pts) LDE DT and Block Diag (Lec 3, OW 3.2, Arcak Lec 2)
An LTI system with scalar input \( u[n] \) (here \( u[\cdot] \) is not the unit step) and scalar output \( y[n] \) is described by the state-space equations: \( x[n+1] = Ax[n] + Bu[n] \) and output equation \( y[n] = Cx[n] + Du[n] \).

a. For each \( \{A,B,C,D\} \) below: draw the block diagram realization of this system using multiply by constant, summation blocks, and the minimum number of delay blocks.

b. Write the corresponding difference equation:
\[
\sum_{k=0}^{N} a_k y[n-k] = \sum_{l=0}^{N} b_l u[n-l]
\]

c. With initial conditions \( y[n] = 0 \) for \( n < 0 \), find the unit sample response (that is, for \( u[n] = \delta[n] \)) for \( 0 \leq n \leq 5 \) (by hand is ok).

i. 
\[
A = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
-1/2 & 1 & 1/2
\end{bmatrix}
\quad B = \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\quad C = [-1/2 1 1/2] 
\quad D = [1]
\]

ii. 
\[
A = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{bmatrix}
\quad B = \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\quad C = [1/2 -1 -1/2] 
\quad D = [1]
\]

2. (24 pts) Fourier Series and Fourier Transform (OW 3.3, 3.5, 4.4)
Calculate the Fourier Series for the following signals. That is, find the complex scaling coefficients \( a_k \) and fundamental frequency \( \omega_s = \frac{2\pi}{T} \). Sketch the time function and line spectrum (\( a_k \) vs. \( \omega = k\omega_s \)) for each signal. Note that \( \delta(at) = \frac{1}{a} \delta(t) \).

a. \( x_1(t) = 2\Pi(t) * \Pi(t) * \frac{1}{2} \text{comb}(t/2) \)

b. \( x_2(t) = 4\Pi(t) * \Pi(t) * \frac{1}{4} \text{comb}(t/4) \)

c. \( x_3(t) = 8\Pi(t) * \Pi(t) * \frac{1}{8} \text{comb}(t/8) \)

d. Compare the line spectra above to the Fourier transform \( \mathcal{F}\{\Pi(t) * \Pi(t)\} \) (compare sketches).

3. (20 pts) Convolution and Fourier Transform (OW 3.8, 3.10, 4.2, 4.4)
This problem examines filtering of a periodic signal from time domain and frequency domain approaches. Given signal \( x(t) = \Pi(t) * \frac{1}{8} \text{comb}(t/8) \), and LTI low pass filter with impulse response \( h(t) = e^{-t}u(t) \).

a. Find \( y_1(t) = x(t) * h(t) \) by convolution and include sketches.

b. Find the Fourier transforms \( \mathcal{F}\{x(t)\} = X(j\omega), \mathcal{F}\{h(t)\} = H(j\omega) \), and \( Y_2(j\omega) = H(j\omega)X(j\omega) \) using Fourier Transform properties and Tables 4.1 and 4.2 as appropriate.

c. Sketch the magnitude of the Fourier Transforms \( X(j\omega), H(j\omega), Y_2(j\omega) \).

d. Explain why \( \mathcal{F}\{y_1(t)\} \) and \( Y_2(j\omega) \) are the same, considering the Fourier series for \( y_1(t) \).
4. (18 pts) Fourier Transform (OW 4.4, 4.6, Lec 6)

For \( x(t) = \frac{\sin 150\pi t}{\pi t} \) and \( h(t) = \frac{\sin 200\pi t}{\pi t} - \frac{\sin 100\pi t}{\pi t} \),

find and sketch the Fourier Transform (real and imaginary parts) of:

a. \( x(t)h(t) \)  

b. \( x(t)\cos(2\pi 10^2 t) \)  

c. \( x(t) \ast h(t) \)  

d. \( x^2(t) \)  

e. \( x(t) \ast \cos(2\pi 10^2 t) \)  

f. \([x(t)\cos(2\pi 10^2 t)] \ast h(t)\)  

5. (16 pts) Fourier Transform (OW 4.3, 4.6, Lec 6)

a. Show that any signal \( x(t) \) can be written as

\( x(t) = x_e(t) + x_o(t) \)

where \( x_e(t) \) is even symmetric: \( x_e(-t) = x_e(t) \), and \( x_o(t) \) is odd symmetric: \( x_o(-t) = -x_o(t) \).

b. Assuming \( x(t) \) is real, show that

\( \mathcal{F}\{x_e(t)\} = \text{Re}\{X(j\omega)\} \) and \( \mathcal{F}\{x_o(t)\} = j\text{Im}\{X(j\omega)\} \).

c. Find \( x_e(t) \) and \( x_o(t) \) and calculate their Fourier transforms for \( x(t) = t\Pi(t - 1/2) \).