

The LDE is given by:

$$\sum_{k=0}^N a_k y[n-k] = b_0 \delta[n] \quad \text{or} \quad y[n] = \frac{1}{a_0} \left( - \sum_{k=1}^N a_k y[n-k] + b_0 \delta[n] \right) \quad (1)$$

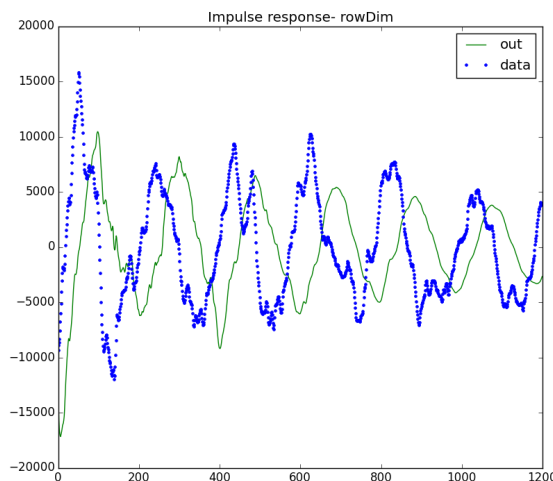
Assume initial conditions  $y[n] = 0$  for  $n < 0$ . Let  $a_0 = 1$ .

1. (10 pts) Assume  $N = 4$  and given coefficients  $a_1, a_2, a_3, a_4$ . Find  $y[n]$  for  $n = 0, 1, 2, 3$  for unit sample input (note that  $y[0] = b_0/a_0$ ).

$$\begin{aligned} y[0] &= b_0 \delta[0] \\ y[1] &= -a_1 y[0] \\ &= -a_1 b_0 \\ y[2] &= -a_1 y[1] - a_2 y[0] \\ &= a_1 a_1 b_0 - a_2 b_0 \\ y[3] &= -a_1 y[2] - a_2 y[1] - a_3 y[0] \\ &= -a_1^3 b_0 + a_1 a_2 b_0 + a_1 a_2 b_0 - a_3 b_0 \end{aligned}$$

2. (30 pts) In the PY1-drum notebook, add code to the for loop to calculate  $z$  which is  $y[n]$  from eq.(1). Also, update `state[]` which should contain  $[y[n] \ y[n-1] \ \dots \ y[n-N]]^T$ .

```
z = np.dot(-a, state) # calculate next state
state[1:colDim] = state[0:colDim-1] # update state vector
state[0] = z
out[i] = z # store value for plotting
```



3. (30 pts) In the PY1-drum notebook, choose the size of the state vector `colDim` and number of data samples `rowDim` such that rms error is less than 2500. Note that data is sampled at 44 kHz, so samples are only 22  $\mu$ s apart.

```
colDim=400 # choose number of states
```

```
rowDim = 1200 # choose number of samples
```

This gives RMS error of 2363.82. Note `rowDim` gives about 27 ms of sound, which appears to show some of the envelope decay over about 6 cycles of the data. The LDE order `colDim` is large enough to provide some shape details, which improves the sound quality.

**4. (20 pts)** Play the `synthDrum.wav` file. Qualitatively describe differences between the sound of the original sampled signal and the reconstructed signal. What might be missing in the reconstructed sound?

The synthesized version seems to be missing some of the time varying tone heard from the drum (it sounds “tinny” rather than a rich complex tone). The initial attack seems sharper compared to the original signal (probably due to the impulse input approximation). Probably in the real drum, the response is more similar to an LTI system as the amplitude decays. Here, LTI is assumed everywhere.

**5. (10 pts)** Choose a different percussion sound file from [freewavesamples.com/sample-type/drums/kick](http://freewavesamples.com/sample-type/drums/kick), choose `rowDim`, `colDim` to find a low rms error. Qualitatively describe whether the LDE output is a good match for the original sound. Note sound file source, and show plot comparing  $y, \hat{y}$  over number of data samples.

Other sound files may not work as well- having a stable filter is necessary here.