Due at 4 pm, Fri. Oct. 14 in HW box under stairs (1st floor Cory) MT Wed Oct 26 4-6 pm.
Reading: O&W Ch 6. DFT Handout.

Note: $\Pi(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$, and $comb(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$.

1. (25 pts) Sampling, Reconstruction, and Interpolation (Lec 8, OW 4.5)
A signal $x(t) = \cos(4\pi t)$ is sampled at 8 Hz:

$$\tilde{x}(t) = x(t) \cdot \sum_{n=-\infty}^{n=\infty} \delta(t - \frac{n}{8}) \quad (1)$$

a) Sketch $X(j\omega)$ and $\tilde{X}(j\omega)$.
b) Find and sketch $X(e^{j\Omega})$ for $x[n] = \cos(\frac{\pi n}{2})$ and compare to $X(j\omega)$ and $\tilde{X}(j\omega)$ from part a.
c) Find an ideal reconstruction filter $R(j\omega)$ such that $\hat{X}(j\omega) = R(j\omega)\tilde{X}(j\omega)$ and show that the reconstructed signal $\hat{x}(t)$ is identical to $x(t)$.
d) In the time domain, find an expression for $\hat{x}(t)$ (in terms of $\tilde{x}(t)$), and evaluate $\hat{x}(t = \frac{1}{16})$, either in closed form or numerically. (Note that the reconstruction filter interpolates between samples to find this value. If the signal is bandlimited, the ideal LPF does exact interpolation.)

2. (15 pts) DFT warmup (DFT H.O. Lec. 11,12, Arcak 9)
Given continuous time signal $x(t) = \cos(1000\pi t)$. This signal is sampled with $T_s = 200\mu s$ for $N = 100$ samples.
a. Find $x[n]$, and $X[k]$, the DFT of $x[n]$. (Hint: express $x[n]$ in terms of $e^{j\omega_o n}$.)
b. What is the spacing of the samples in the frequency domain? (For example, what frequency does $k = 10$ correspond to?)
c. Approximately sketch $X[k]$.

3. (40 pts) DFT (DFT H.O. Lec. 11,12, Arcak 9)
Consider the signal flow diagram shown in Figure 1. For each window $w(t)$, signal $x(t)$, and sampling combination below, sketch $x(t), x_w(t), x_\delta(t), x'(t)$ and their magnitude spectra. Also sketch magnitude and phase for $X[k]$ (derived from $X'(j\omega)$).
Let $T_o = 16T_s$, $T_s = \frac{1}{2}$ sec, $x(t) = \cos(\pi t/2)$.

i. Let $w(t) = \Pi(\frac{t}{T_o})$
ii. Let $w(t) = \Pi(\frac{2t}{T_o})$
iii. Let $w(t) = \Pi(\frac{4t}{T_o})$
iv. Let $w(t) = \Pi(\frac{t-1}{T_o})$ *changed from*: $\Pi(\frac{t-\frac{T_o}{2}}{T_o})$

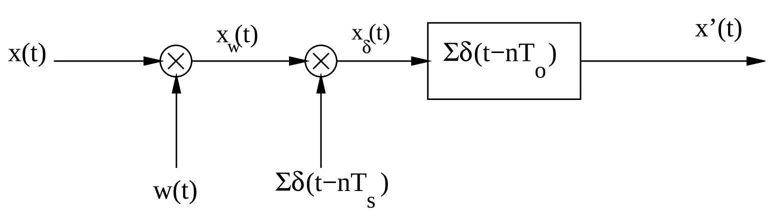
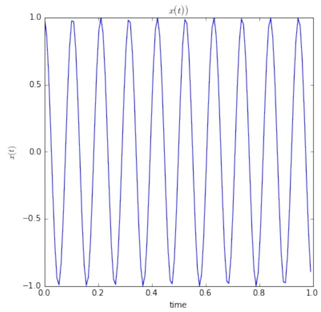


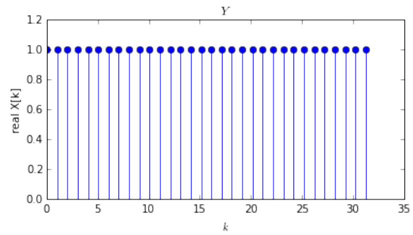
Figure 1: Window, sample in time, sample in frequency (DFT)

4. (20 pts) DFT (Lec. 11,12, DFT H.O.)

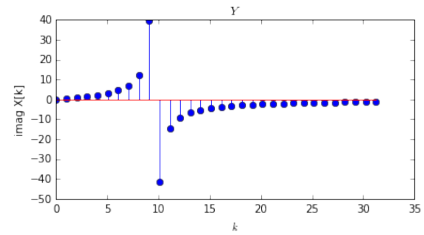
A real signal $x(t) = \cos(2\pi 9.5t)$, $(\omega_o = 2\pi 9.5)$ is sampled with $N = 128$ for 1 second. The DFT of $x[n]$ is calculated using `X = np.fft.fft(x)`. The DFT of the signal is shown below for samples $X[0]...X[31]$. Using reasoning as in problem 3iv above, explain the differences between the DFT of $x[n]$ and $X(j\omega)$, the FT of $x(t) = \cos(\omega_o t)$. In particular, consider the effects on $X'(j\omega))$ of the window and time shift.



$x[n]$



Re $X[k]$



Im $X[k]$