Professor Fearing EECS120/Problem Set 9 v 1.01 Fall 2016

Due at 4 pm, Fri. Nov. 4 in HW box under stairs (1st floor Cory)

Note: $\Pi(t) = u(t+\frac{1}{2}) - u(t-\frac{1}{2})$, and $comb(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$.

1. (30 pts) Lec15 OW Ch. 7, Up/Down sample handout

A sampled signal $x[n] = \frac{2\sin(\pi n/2)}{\pi n}$ is upsampled by 3, low-pass filtered, and then down sampled by 2 to obtain a new signal $x_b[n]$ which is equivalent to the original signal sampled at 1.5 times faster sample rate. Following the handout and noting heights and widths, sketch the pairs:
$x[n] \leftrightarrow X(e^{j\Omega})$, $x_p[n] \leftrightarrow X_p(e^{j\Omega})$, $x_u[n] \leftrightarrow X_u(e^{j\Omega})$, $x_d[n] \leftrightarrow X_d(e^{j\Omega})$, and $x_b[n] \leftrightarrow X_b(e^{j\Omega})$.
Also specify height of $H(e^{j\Omega})$ (note: should be Ω in LPF block).

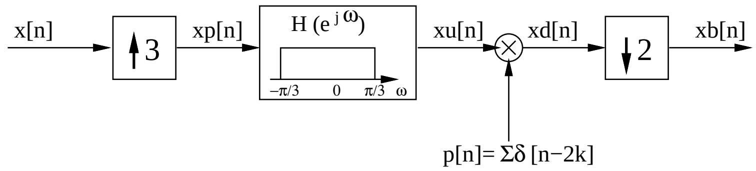

Fig.1.

2. (20 pts) Sampling and Reconstruction (OW 7.1)

In many cases of conversion from discrete to continuous time, we considered an ideal low-pass filter for reconstruction. In practice, a digital-to-analog converter, which can be modelled as a zero-order hold, is followed by a low-complexity analog filter. Consider the system shown in Fig. 2, where $x(t) = \cos(400\pi t), T_{s1} = \frac{1}{600}$ sec, $T = T_s$, and $h(t) = 400\pi e^{-400\pi t}u(t)$.

a. Sketch $x(t), x_\delta(t), x_z(t), x_r(t)$ (for $0 \le t \le 6T_{s1}$) and associated magnitude spectra. ($x_r(t)$ will be an approximate sketch.)

b. Numerically estimate the fraction of the power in $x_r(t)$ which is not at the original signal frequency of 200 Hz.

c. With upsampling, a discrete time signal can be be generated with a higher effective sampling rate (if the original signal is bandlimited to less than half the original sampling frequency). Consider now $T_{s2} = \frac{1}{10}T_{s1}$, and $T = T_{s2}$. Sketch the new $x_{r2}(t)$ and $|X_{r2}(j\omega)|$ and numerically estimate the fraction of the power in $x_r(t)$ which is not at the original signal frequency of 200 Hz.

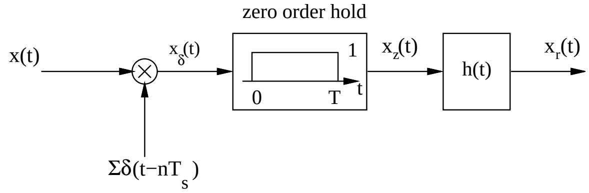

Fig. 2. Sampling and zero order hold.

3. (20 pts) DFT H.O., Up/down sample H.O.

This problem considers up and down sampling with DFT instead of DTFT. Consider a 1 kHz cosine sampled at 8 kHz for 32 samples. (Assume $x[0] = 1$.)

a) Determine $X[k]$ the DFT of $x[n]$. What is the spacing of the samples in the frequency domain? For example, what frequency does k=3 correspond to?

b) Create a new sequence $v[n]$ of length 16 by downsampling $x[n]$, and sketch $v[n]$. That is $v[n] = x[2n]$. Determine and sketch $V[k]$ the 16 point DFT of $v[n]$. What is the new spacing of samples in the frequency domain?

c) Create a new sequence $y[n]$ of length 64 by upsampling $v[n]$. That is $y[n] = v[n/4]$ for n a multiple of 4 and $y[n] = 0$ for n otherwise. Determine $Y[k]$ the 64 point DFT of $y[n]$. What is the new spacing of samples in the frequency domain?

d) A filter $H[k]$ can be used to interpolate between values of $y[n]$ to obtain $z[n]$, that is $Z[k] = H[k]Y[k]$. Sketch $H[k], Z[k]$ and $z[n]$ the inverse DFT of $Z[k]$. What is the equivalent spacing of samples in time domain?

4. (10 pts) Region of convergence, pole/zero diagram (Lec 16,17 OW 9.2)

For each part below, $y(t)$ is the output for an LTI system with impulse response $h(t)$ and input $x(t)$. Show the pole and zero locations, and the region of convergence in the $\sigma - j\omega$ plane for each $Y(s)$, using the Laplace transform integral.

i. $x(t) = te^{-2t}u(t), h(t) = e^{+2t}u(t)$

ii. $x(t) = e^{-6t}u(t), h(t) = \sin(4\pi t)u(t)$.

5. (20 pts) Laplace Transform Lec 17,18 OW 9.3, 9.5-9.7

For each part below, use Laplace transforms and LTI properties (tables ok) to find the output $y(t)$ for an LTI system with impulse response $h(t)$ and input $x(t)$.

i. $x(t) = e^{-t}u(t)$, $h(t) = u(t)$.

ii. $x(t) = e^{-2t}u(t)$, $h(t) = e^{-3t}u(t-1)$.

iii. $x(t) = u(t)$, $h(t) = \cos(8\pi t)u(t)$

iv. $x(t) = e^{-2t}u(t) + tu(t)$, $h(t) = \delta(t-1) + \dot{\delta}(t-0.5)$

v. $x(t) = \cos(2\pi t)u(t)$, $h(t) = tu(t)$.