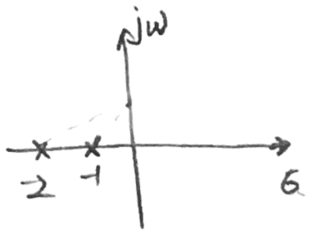
EE 120 GSI: Ming PS 11

prob 1.

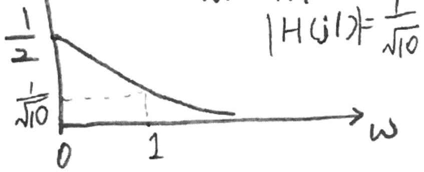
$$H_1(s) = \frac{1}{(s+1)(s+2)}$$

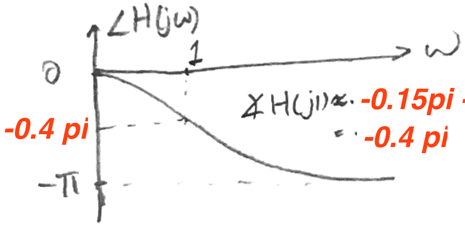


pole-zero plot

$$|H(j\omega)| = \frac{1}{\sqrt{1+\omega^2}\sqrt{4+\omega^2}}$$

$$|H(j1)| = \frac{1}{\sqrt{10}}$$



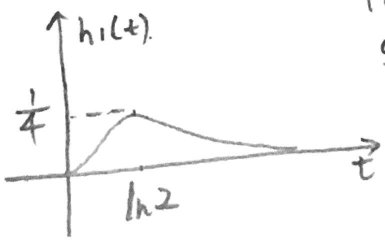


$\measuredangle H(j1) \approx$ -0.15pi - 0.25pi

$\approx$ -0.4 pi

$$H_1(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$$h_1(t) = e^{-t}u(t) - e^{-2t}u(t)$$

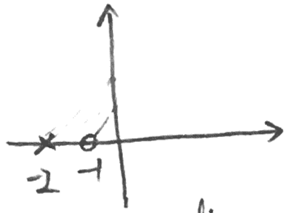


Find the maximum of $h_1(t)$:

$$\frac{dh_1(t)}{dt} = -e^{-t} + 2e^{-2t} = 0$$

$$\Rightarrow t = \ln 2$$

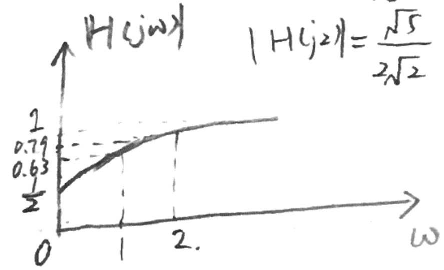
$$H_2(s) = \frac{s+1}{s+2}$$

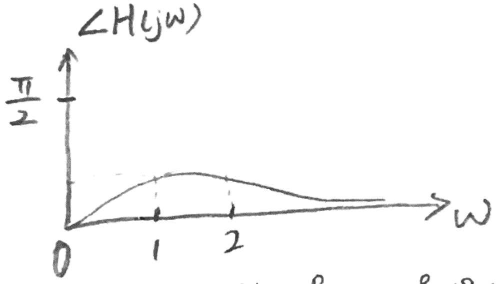


pole-zero diagram

$$|H(j1)| = \frac{\sqrt{2}}{\sqrt{5}} = \sqrt{\frac{2}{5}}$$

$$|H(j2)| = \frac{\sqrt{5}}{2\sqrt{2}}$$



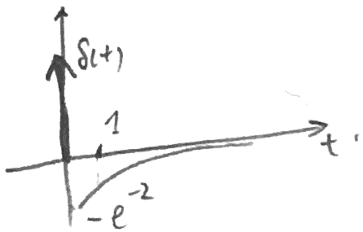


$\measuredangle H(j1) \doteq 45^\circ - 26.6^\circ \doteq 18.4^\circ$

$\measuredangle H(j2) \approx 63^\circ - 45^\circ \approx 18^\circ$

$$H_2(s) = 1 - \frac{1}{s+2}$$

$$h_2(s) = \delta(t) - e^{-2t}u(t)$$



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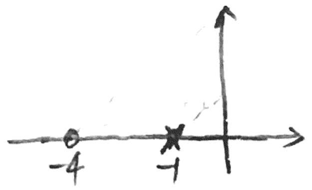
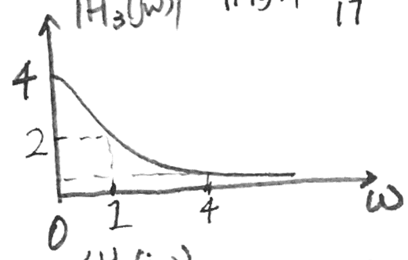
$$H_3(s) = \frac{s+4}{(s+1)^2}$$

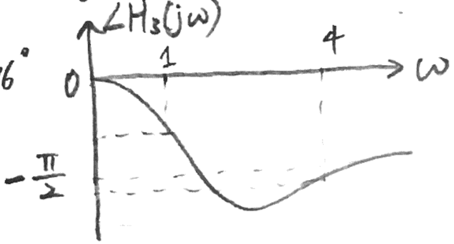


pole-zero diagram

$|H(j1)| = \frac{\sqrt{17}}{2} \approx 2$

$|H(j4)| = \frac{4\sqrt{2}}{17} \approx 0.33$





$\measuredangle H(j1) = 14^\circ - 45^\circ 2 = -76^\circ$

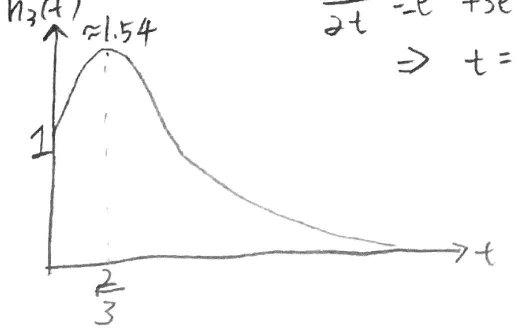
$\measuredangle H(j4) = 45^\circ - 2 \cdot (76^\circ) = -107^\circ$

$$H_3(s) = \frac{s+1+3}{(s+1)^2} = \frac{1}{s+1} + \frac{3}{(s+1)^2}$$

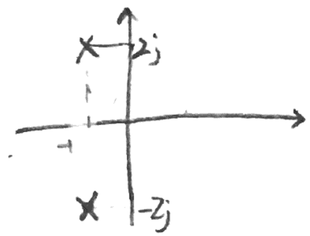
$$h_3(t) = e^{-t}u(t) + 3te^{-t}u(t)$$

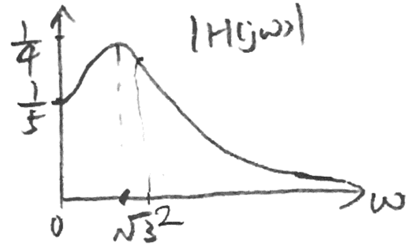
$$\frac{\partial h_3(t)}{\partial t} = e^{-t} + 3e^{-t} - 3te^{-t} = 0$$

$$\Rightarrow t = \frac{2}{3}$$



$$H_4(s) = \frac{1}{s^2+2s+5} = \frac{1}{(s+1+2j)(s+1-2j)}$$



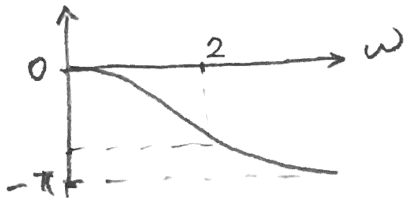


$$|H(j\omega)| = \frac{1}{\sqrt{\omega^4 - 6\omega^2 + 25}} = \frac{1}{\sqrt{(\omega^2-3)^2+16}}$$

$|H(j2)| = \frac{1}{\sqrt{17}} \approx \frac{1}{4}$

maximal at $\omega^2 = 3$ or $\omega = \pm\sqrt{3}$

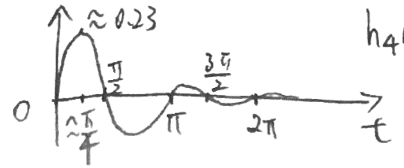
$\measuredangle H(j2) = 0 - 76^\circ = -76^\circ$



$$H_4(s) = \frac{\frac{1}{4}j}{s+1+2j} + \frac{-\frac{1}{4}j}{s+1-2j}$$

$$h_4(t) = \frac{1}{4}je^{-(1+2j)t} - \frac{1}{4}je^{-(1-2j)t}$$

$$= \frac{1}{2}e^{-t}\sin 2t \, u(t)$$



$h_4(t)$

Prob 2

a). Since each pole will cause $|H(j\omega)|$ to decrease at rate $\frac{1}{\omega}$, and each zero will cause $|H(j\omega)|$ to increase at rate ω. When there is an imbalance in the number of poles and zeros, $|H(j\omega)|$ will exhibit either net increase or net decrease. As we see in the problem $|H(j\omega)|$ decreases as $\omega \to \infty$, we cannot have more zeros than poles.

Also, If we have the same # of poles and zeros, then $|H(j\omega)|$ will approach a non-zero constant:

$$\lim_{s\to\infty} K \frac{\prod_{i=1}^{N}(s-z_i)}{\prod_{i=1}^{N}(s-p_i)} = K \neq 0$$

this can't be the case since $|H(j\omega)| \to 0$ as $\omega \to \infty$.

Therefore, we must have more poles than zeros.

b) $$H(s) = \frac{s^k + a_1 s^{k-1} + \cdots + a_k}{s^m + b_1 s^{m-1} + \cdots + b_m}$$

For real time-domain signal the coefficients $a_1 \cdots a_k$, $b_1, \cdots, b_m$ are all real. This neccessitates the poles and zeros to be purely real or appear as conjugate pairs

e.g $(s+\gamma+\beta j)(s+\gamma-\beta j) = (s+\gamma)^2+\beta^2$

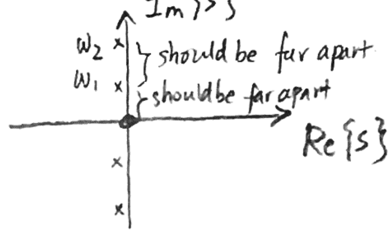
$= s^2+2s\gamma+\gamma^2+\beta^2$ have real coefficents

c). ① $|H(j\omega)|$ is 0 at origin ⇒ one zero at the origin.

② two peaks as ω increases ⇒

two pairs of poles close to the $j\omega$-axis

[reason: if d is the distance between the point $(0, j\omega)$ on the $j\omega$-axis and the pole $(\delta, j\omega_0)$, where δ is small, then d is small, but $\frac{1}{d}$ is large ⇒ peak in $|H(j\omega)|$]

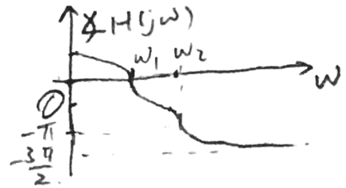


d). For the above system,

$$H(j\omega) = \frac{j\omega}{(j(\omega+\omega_1)+6)(j(\omega-\omega_1)+6)(j(\omega+\omega_2)+6)(j(\omega-\omega_2)+6)}$$

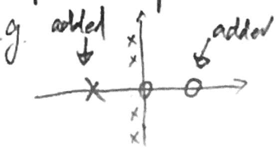
$$\measuredangle H(j\omega) = \tan^{-1}\omega - \tan^{-1}\frac{\omega+\omega_1}{6} - \tan^{-1}\frac{\omega-\omega_1}{6} - \tan^{-1}\frac{\omega+\omega_2}{6} - \tan^{-1}\frac{\omega-\omega_2}{6}$$

$\measuredangle H(j\omega)|_{\omega=0} = 0$. $\measuredangle H(j\omega)|_{\omega\to\infty} = \frac{\pi}{2} - \frac{\pi}{2}\cdot 4 = \frac{-3\pi}{2}$



The phase response is NOT unique for the given $|H(j\omega)|$ since we can add a pole and a zero on both sides of the real axis and still have the same magnitude, but different phase responses.

e.g.



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Prob 3

IVT: $x(0) = \lim_{s \to \infty} sX(s)$

FVT: $x(\infty) = \lim_{s \to 0} sX(s)$

a).

$x(0) = \lim_{s \to \infty} \frac{s^2}{s^2+5s+6} = 1$

$x(\infty) = \lim_{s \to 0} \frac{s^2}{s^2+5s+6} = 0$

b). $x(0) = \lim_{s \to \infty} \frac{s}{s^2+s} = 0$

$x(\infty) = \lim_{s \to 0} \frac{s}{s^2+s} = \frac{1}{2s+1}\Big|_{s=0} = 1$ (L'Hopital)

c).

$x(0) = \lim_{s \to \infty} \frac{s^2-s}{s^2+s} = 1$

$x(\infty) = \lim_{s \to 0} \frac{s^2-s}{s^2+s} = \frac{2s-1}{2s+1}\Big|_{s=0} = -1$

d). $x(0) = \lim_{s \to \infty} \frac{s^2-s}{s^2+5s+6} = 1$

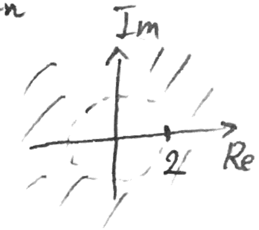
$x(\infty) = \lim_{s \to 0} \frac{s^2-s}{s^2+5s+6} = 0$

Prob 4.

a).

$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=0}^{\infty} 2^n z^{-n}$

$= \sum_{n=0}^{\infty} \left(\frac{2}{z}\right)^n = \frac{1}{1-2z^{-1}}$

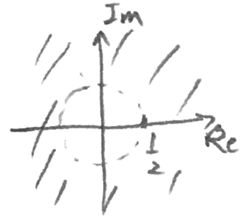


for $|2z^{-1}| < 1$ or $|z| > 2$

ROC : $|z| > 2$

b). $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$

$= \sum_{n=0}^{\infty} \left(\frac{1}{2} z^{-1}\right)^n = \frac{1}{1-\frac{1}{2}z^{-1}}$

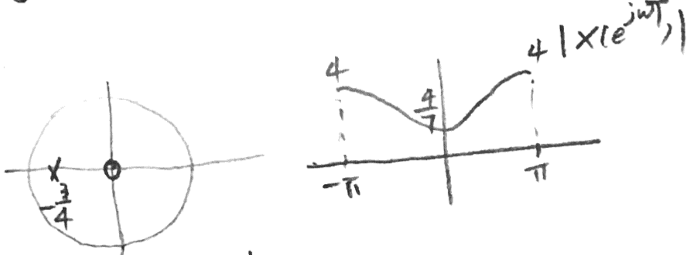


for $|\frac{1}{2}z^{-1}| < 1$ or $|z| > \frac{1}{2}$.

ROC: $|z| > \frac{1}{2}$.

Prob 5.

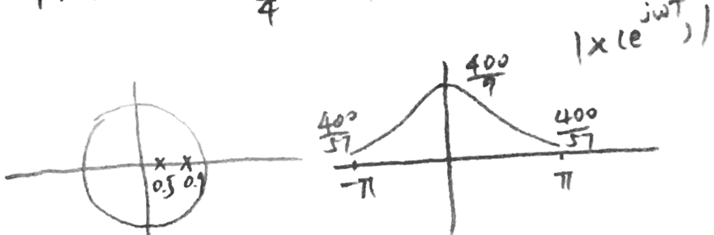
(a)



$$|X(e^{j0})| = \frac{1}{\frac{7}{4}} = \frac{4}{7}$$

$$|X(e^{j\pi})| = \frac{1}{\frac{1}{4}} = 4$$

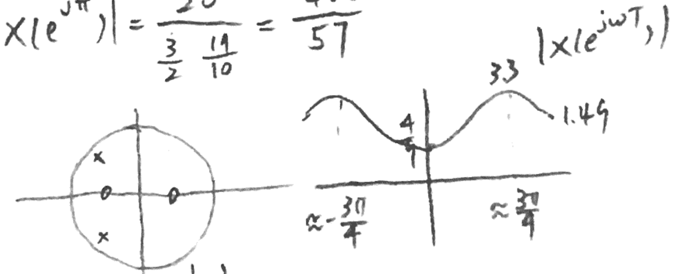
(b).



$$|X(e^{j0})| = \frac{20}{\frac{1}{2}\ \frac{1}{10}} = 400$$

$$|X(e^{j\pi})| = \frac{20}{\frac{3}{2}\ \frac{19}{10}} = \frac{400}{57}$$

(c).



$$|X(e^{j0})| = \frac{\frac{1}{2}\cdot\frac{1}{2}}{\frac{3}{4}\cdot\frac{3}{4}} = \frac{4}{9}$$

$|X(e^{j\frac{3\pi}{4}})| \approx 3.3$, $|X(e^{j\pi})| \approx 1.49$

Prob 6.

IVT: $x(0) = \lim_{z\to\infty} X(z)$

FVT: $x(\infty) = \lim_{z\to 1} X(z)(1-z^{-1})$

(a) $x(0) = \lim_{z\to\infty} \frac{z}{z+\frac{3}{4}} = 1$

$x(\infty) = \lim_{z\to 1} \frac{z}{z+\frac{3}{4}}(1-z^{-1}) = 0$

(b) $x(0) = \lim_{z\to\infty} \frac{20}{(z-0.5)(z-1)} = 0$

$x(\infty) = \lim_{z\to 1} \frac{20}{(z-0.5)(z-1)}(1-z^{-1}) = 40$

(c). $x(0) = \lim_{z\to\infty} \frac{(z-\frac{1}{2})(z+\frac{1}{2})}{(z+\frac{3}{4}e^{j\pi/4})(z+\frac{3}{4}e^{-j\pi/4})} = 1$

$x(\infty) = \lim_{z\to 1} X(z)(1-z^{-1}) = 0$

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