University of California at Berkeley
College of Engineering
Department of Electrical Engineering and Computer Sciences

EECS 120: Signals and Systems
Fall Semester 1998

Examples of LTI Filtering of Periodic Signals

$$x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{jk\omega_0 t} \longrightarrow \boxed{\begin{matrix}\text{LTI System} \\ h(t) \leftrightarrow H(j\omega)\end{matrix}} \longrightarrow y(t) = \sum_{k=-\infty}^{\infty} Y[k]e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} X[k]H(jk\omega_0)e^{jk\omega_0 t}$$

Note: $|Y[k]| = |X[k]| \cdot |H(jk\omega_0)|$ and $\arg(Y[k]) = \arg(X[k]) + \arg(H(jk\omega_0))$

Input Signal:

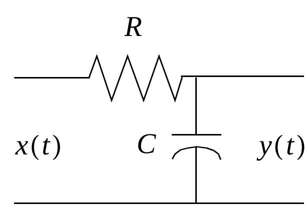
$x(t)$ that is a square wave, which has Fourier series coefficients: $X[k] = \frac{2T_s}{T}\operatorname{sinc}\left(k\frac{2T_s}{T}\right)$.

We choose $\frac{T_s}{T} = \frac{1}{8}$, so that $X[k] = \frac{1}{4}\operatorname{sinc}\left(\frac{k}{4}\right)$.

Fourier series representation: $x(t) = \sum_{k=-\infty}^{\infty} \frac{1}{4}\operatorname{sinc}\left(\frac{k}{4}\right)e^{jk\omega_0 t}$, where $\omega_0 = \frac{2\pi}{T}$.

LTI Systems:

1. First-Order Lowpass Filter:



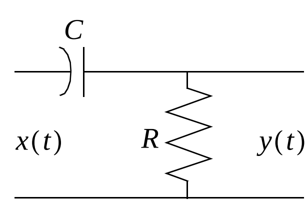
$h(t) = \frac{1}{RC}e^{-t/RC}u(t)$, $H(j\omega) = \frac{1}{1 + j\omega RC}$.

$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$, $\arg(H(j\omega)) = -\tan^{-1}(\omega RC)$.

Fourier series representation of output:

$$y(t) = \sum_{k=-\infty}^{\infty} X[k]\frac{1}{1 + jk\omega_0 RC}e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} \frac{1}{4}\operatorname{sinc}\left(\frac{k}{4}\right)\frac{1}{1 + jk\omega_0 RC}e^{jk\omega_0 t}.$$

2. First-Order Highpass Filter:



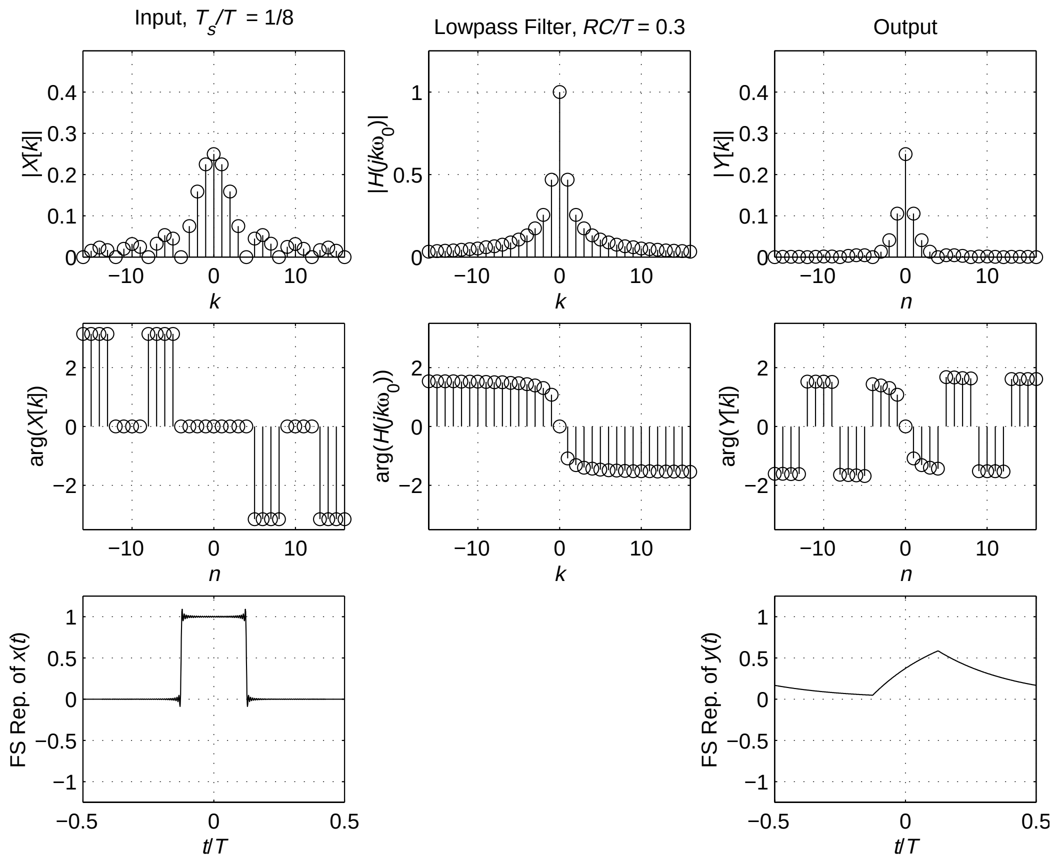
$h(t) = \delta(t) - \frac{1}{RC}e^{-t/RC}u(t)$, $H(j\omega) = \frac{j\omega RC}{1 + j\omega RC}$.

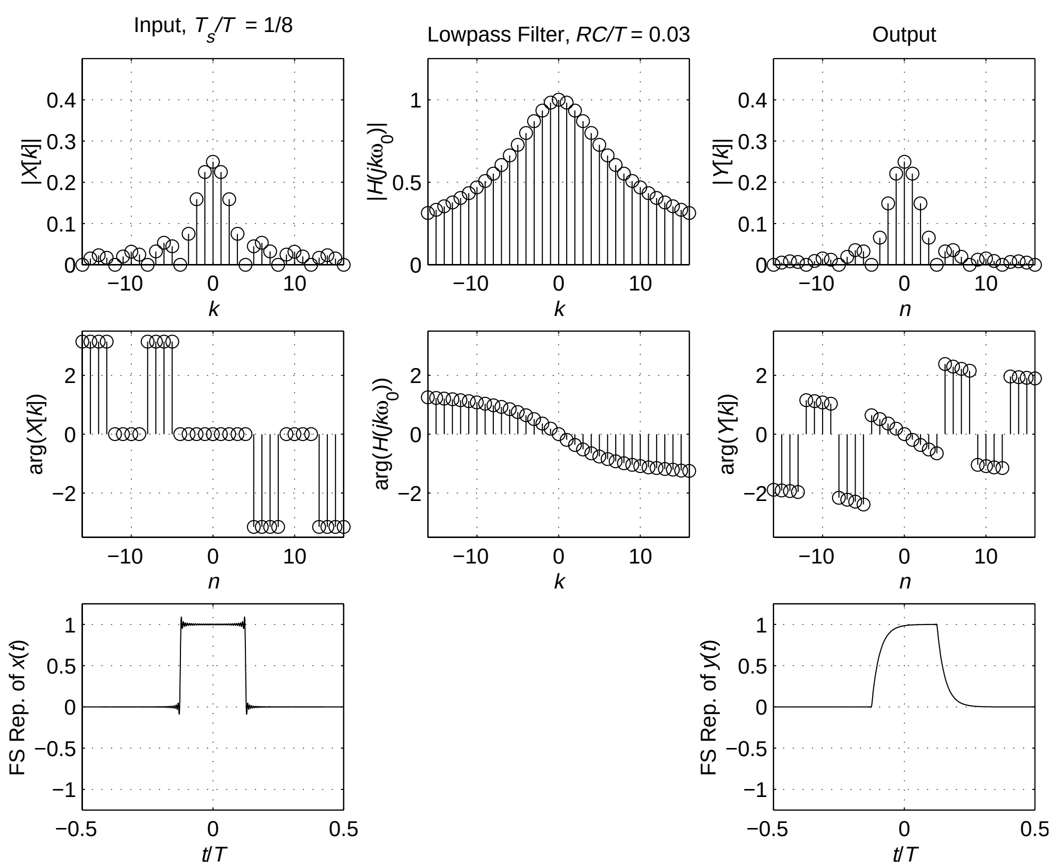
$|H(j\omega)| = \frac{|\omega|RC}{\sqrt{1 + (\omega RC)^2}}$, $\arg(H(j\omega)) = \frac{\pi}{2}\operatorname{sgn}\omega - \tan^{-1}(\omega RC)$.

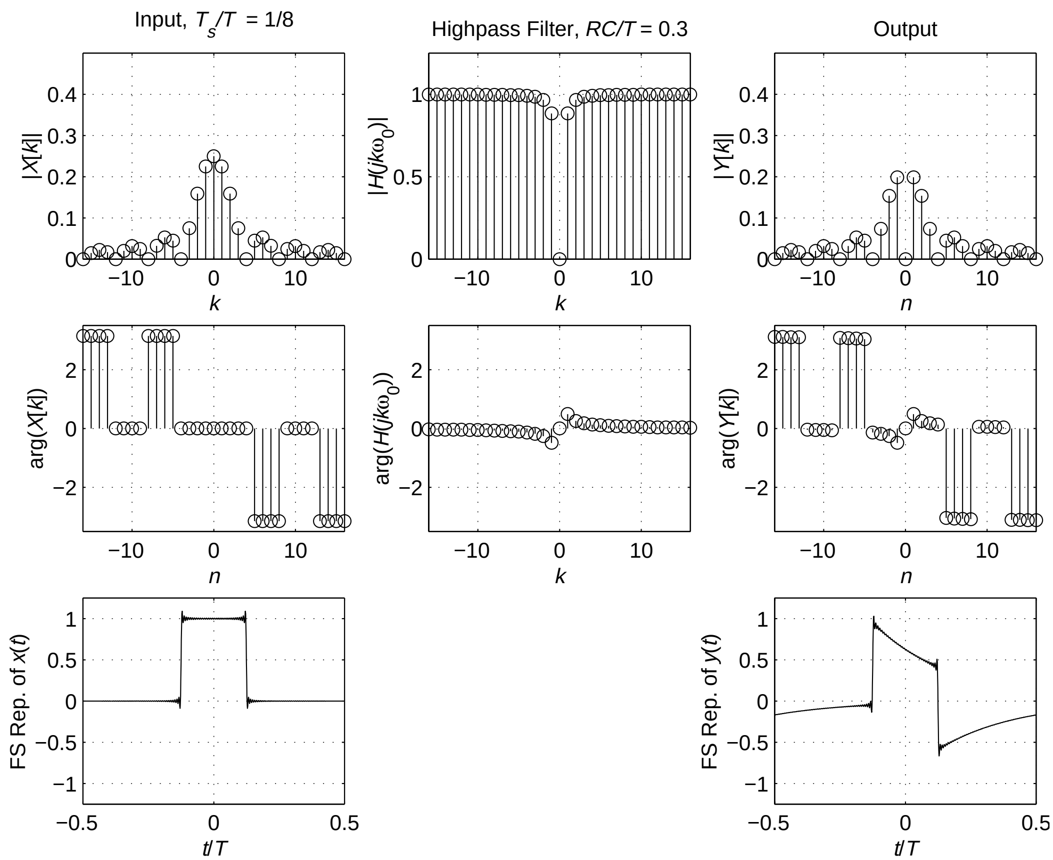
Fourier series representation of output:

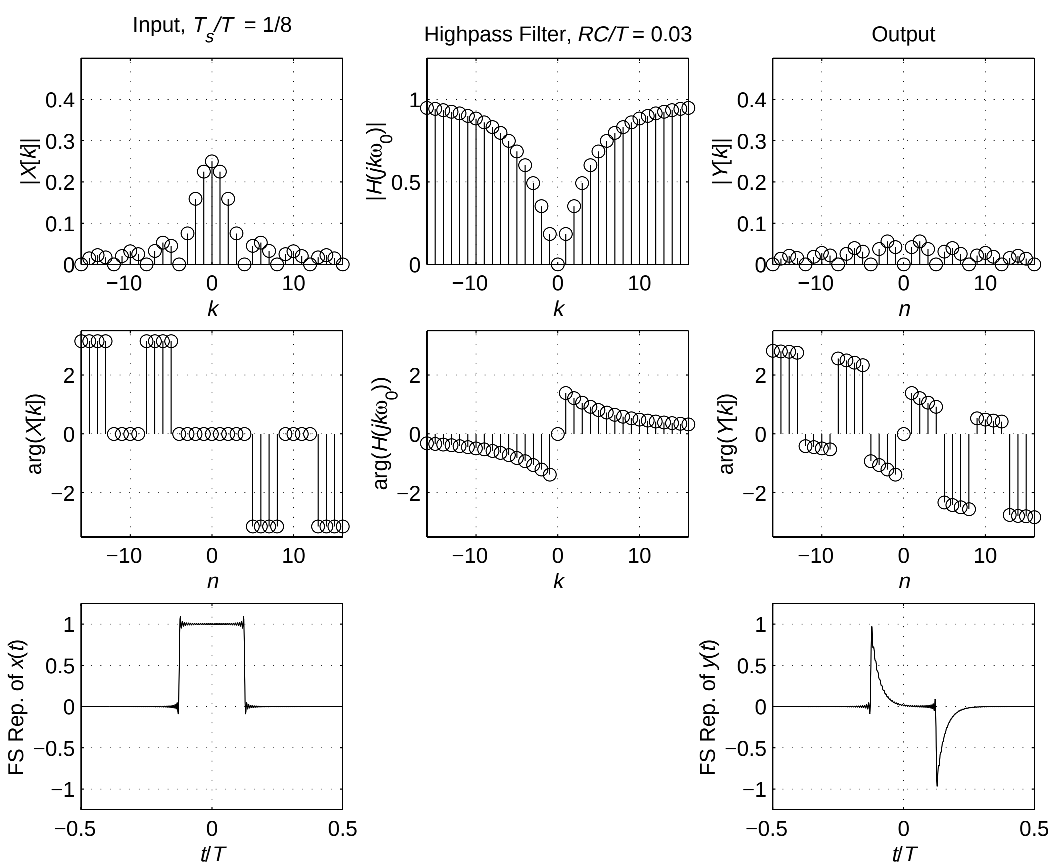
$$y(t) = \sum_{k=-\infty}^{\infty} X[k]\frac{jk\omega_0 RC}{1 + jk\omega_0 RC}e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} \frac{1}{4}\operatorname{sinc}\left(\frac{k}{4}\right)\frac{jk\omega_0 RC}{1 + jk\omega_0 RC}e^{jk\omega_0 t}.$$

The plots below of $x(t)$ and $y(t)$ include terms from $k = -128$ to 128.


Input, T_s/T = 1/8
Lowpass Filter, RC/T = 0.3
Output
$|X[k]|$
$|H(jk\omega_0)|$
$|Y[k]|$
arg($X[k]$)
arg($H(jk\omega_0)$)
arg($Y[k]$)
FS Rep. of $x(t)$
FS Rep. of $y(t)$
k
n
t/T


Input, T_s/T = 1/8
Lowpass Filter, RC/T = 0.03
Output
$|X[k]|$
$|H(jk\omega_0)|$
$|Y[k]|$
arg($X[k]$)
arg($H(jk\omega_0)$)
arg($Y[k]$)
FS Rep. of $x(t)$
FS Rep. of $y(t)$
k
n
t/T


Input, T_s/T = 1/8
Highpass Filter, RC/T = 0.3
Output
$|X[k]|$
$|H(jk\omega_0)|$
$|Y[k]|$
arg($X[k]$)
arg($H(jk\omega_0)$)
arg($Y[k]$)
FS Rep. of $x(t)$
FS Rep. of $y(t)$
k
n
t/T


Input, T_s/T = 1/8
Highpass Filter, RC/T = 0.03
Output
$|X[k]|$
$|H(jk\omega_0)|$
$|Y[k]|$
arg($X[k]$)
arg($H(jk\omega_0)$)
arg($Y[k]$)
FS Rep. of $x(t)$
FS Rep. of $y(t)$
k
n
t/T