University of California at Berkeley
College of Engineering
Department of Electrical Engineering and Computer Sciences

EECS 120: Signals and Systems
Fall Semester 1998

Prerequisites for EECS 120

EECS 120 requires Math 53 and Math 54, and either EECS 20 or EECS 40, as prerequisites. In particular, the following subjects are heavily used in the course. Go through the review questions below to make sure you can work them.

Integration: know how to integrate functions of one variable, especially integrals with infinite limits, and those involving *e*. Be familiar with change of variables and integration by parts.

Complex numbers: understand both rectangular and polar representations. Be able to convert from one form to another and to perform basic arithmetic operations on complex numbers.

Trigonometry: be familiar with identities for powers of sines and cosines, and sines and cosines of the sum and difference of two angles. Know the complex-exponential representations of sine and cosine functions. Be familiar with Euler's formula.

Linear Algebra: be able to add, multiply, invert and transpose matrices. Be comfortable solving systems of linear equations.

Differential Equations: be able to solve simple first- and second-order linear differential equations.

Prerequisite Test

Linear Algebra

1. Consider the following matrices:

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 5 & 8 \\ 3 & 6 & 9 \\ 4 & 7 & 1 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \end{bmatrix}.$$

   a. Invert A and B.

   b. Calculate AX. How many solutions does $AX = 0$ have?

Integration

2. Evaluate the following integrals:

   a. $\int_{-\infty}^{\infty} \delta(t)dt$, where $\delta(t)$ is the unit impulse function.

   b. $\int_{-\infty}^{\infty} t^3 dt$.

   c. $\int_{-\infty}^{\infty} e^{-\alpha t}u(t)dt$, where $u(t)$ is the unit step function.

Complex Numbers

3. Answer the following:

a. What is $3e^{j\frac{\pi}{5}}$ in rectangular form?

b. What is $\frac{7+6j}{2+j}$ in polar form?

Trigonometry

4. Show the following:

a. Prove $\sin^2 x + \cos^2 x = 1$ by using complex exponentials.

b. Prove $\sin^2 x + \cos^2 x = 1$ by use of the relations $\cos^2 x = \frac{1+\cos 2x}{2}$ and $\sin^2 x = \frac{1-\cos 2x}{2}$.

c. State Euler's theorem and explain in one sentence how it is used in phasor analysis.

Differential Equations

5. Solve the following differential equation, assuming zero initial conditions, and assuming that $y(t) = w(t) = 0$ for $t < 0$:

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = \frac{dw}{dt} + 2w .$$

Solutions

1.

a. The inverse of a matrix A is $A^{-1} = \frac{1}{\det(A)}\operatorname{adj}(A)$, where $\det(A)$ is the determinant of A and $\operatorname{adj}(A)$ is the adjoint matrix.

$$\det(A) = 2 \cdot 5 - 3 \cdot 4 = -2$$

$$\operatorname{adj}(A) = \begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix}^t = \begin{bmatrix} 5 & -4 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{5}{2} & 2 \\ \frac{3}{2} & -1 \end{bmatrix}$$

$$\det(B) = 2(6-63) - 5(3-36) + 8(21-24) = 27$$

$$\operatorname{adj}(B) = \begin{bmatrix} -57 & 51 & -3 \\ 33 & -30 & 6 \\ -3 & 6 & -3 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} -\frac{19}{9} & \frac{17}{9} & -\frac{1}{9} \\ \frac{11}{9} & -\frac{10}{9} & \frac{2}{9} \\ -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} \end{bmatrix}$$

b.

$$AX = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x+4y \\ 3x+5y \end{bmatrix}.$$

$AX = 0$ has only one solution: $X = 0$. In fact, for any 2×1 matrix *Y*, $AX = Y$ has only one solution: $X = A^{-1}Y$.

2.

a. $\int_{-\infty}^{\infty} \delta(t)dt = 1$ by the definition of $\delta(t)$.

b. Doing it the hard way:

$$\int_{-\infty}^{\infty} t^3 dt = \left.\frac{t^4}{4}\right|_{-\infty}^{\infty} = 0$$

Or, you can realize that $\int_{-\infty}^{\infty} (\text{any even function of } t)dt = 0$.

c. Since

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases},$$

$$\int_{-\infty}^{\infty} e^{-\alpha t}u(t)dt = \int_{0}^{\infty} e^{-\alpha t}dt = \left.-\frac{1}{\alpha}e^{-\alpha t}\right|_{0}^{\infty} = \frac{1}{\alpha}$$

3.

a.

$$3e^{j\frac{\pi}{5}} = x + jy$$

$$x = 3\cos\frac{\pi}{5} = 1.88,\ y = 3\sin\frac{\pi}{5} = 1.76$$

b.

$$z = \frac{7+6j}{2+j} = \frac{(7+6j)(2-j)}{(2+j)(2-j)} = \frac{20+5j}{5} = 4+j$$

$$|z| = \sqrt{4^2+1^2} = \sqrt{17},\ \angle z = \tan^{-1}\left(\frac{1}{4}\right).$$

Alternative approach:

$$|z| = \frac{|7+6j|}{|2+j|} = \frac{\sqrt{49+36}}{\sqrt{4+1}} = \sqrt{17}$$

$$\angle z = \angle(7+6j) - \angle(2+j) = \tan^{-1}\left(\frac{6}{7}\right) - \tan^{-1}\left(\frac{1}{2}\right),$$

which can be shown to equal $\tan^{-1}\left(\frac{1}{4}\right)$.

4.

a. Recall that

$$\cos x = \frac{1}{2}(e^{jx} + e^{-jx})$$

$$\sin x = \frac{1}{2j}(e^{jx} - e^{-jx})$$

Using this,

$$\sin^2 x + \cos^2 x = \left[\frac{1}{2}(e^{jx} + e^{-jx})\right]^2 + \left[\frac{1}{2j}(e^{jx} - e^{-jx})\right]^2$$

$$= \frac{1}{4} + \frac{1}{4} - \frac{1}{4}e^{2jx} - \frac{1}{4}e^{-2jx} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}e^{2jx} + \frac{1}{4}e^{-2jx} = 1$$

b. Using

$$\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x,\ \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x,$$

$$\sin^2 x + \cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x + \frac{1}{2} - \frac{1}{2}\cos 2x = 1$$

c. Euler's theorem says that

$$e^{jx} = \cos x + j\sin x\ .$$

This allows us to take the real part of a complex exponential and obtain a cosine, meaning that we can treat a sinusoid as a complex exponential to simplify a calculation, and then take the real part at the end of the calculation to get back a sinusoid.

5. Taking the Laplace transform of the differential equation and assuming zero initial conditions:

$$s^2 Y(s) + 4sY(s) + 3Y(s) = sW(s) + 2W(s)\ ,$$

where $Y(s)$ and $W(s)$ are the Laplace transforms of $y(t)$ and $w(t)$, respectively. Solving for $Y(s)$:

$$Y(s) = \frac{s+2}{s^2 + 4s + 3}W(s) = \frac{s+2}{(s+1)(s+3)}W(s)$$

Doing a partial-fraction expansion:

$$\frac{s+2}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{s+3},\ A = \left.\frac{s+2}{s+3}\right|_{s=-1} = \frac{1}{2},\ B = \left.\frac{s+2}{s+1}\right|_{s=-3} = \frac{1}{2}.$$

Inverting $Y(s)$ and using the convolution property of Laplace transforms:

$$y(t) = \left[\frac{1}{2}(e^{-t} + e^{-3t})u(t)\right] \otimes w(t),$$

where the symbol $\otimes$ denotes convolution.