Chapter 4

Digital communication

A digital signal is a discrete-time binary signal $m : \text{Integers} \rightarrow \text{Bin} = \{0, 1\}$. To transmit such a signal it must first be transformed into a baseband analog signal. The baseband signal is then transmitted as such or modulated using techniques of Chapter 3. Many schemes can be modeled as shown in figure 4.1.

![Fig 4.1](image)

Figure 4.1: The binary signal $m$ is converted into an impulse train and then into an analog baseband signal $x$ which is then modulated.

The binary sequence $m$ is divided into $L$-bit blocks or symbols. Each of the $K = 2^L$ symbols is mapped into an amplitude. (The figure shows the case $L = 3$.) If the time between two bits is $T_b$ sec (the bit rate is $R_b = T_b^{-1}$ bits/sec), the time between two symbols is $T = LT_b$. The symbol or baud rate is $R = T^{-1} = L^{-1}R_b$ baud/sec. The resulting symbol sequence $\{a(k)\}$ modulates a train of pulses of the same shape $p$. This can be represented as a convolution of $\sum_n a(n)\delta(t - nT)$ and
the impulse response \( p \). The resulting analog signal

\[
\forall t, \quad x(t) = \sum_k a(k) p(t - kT)
\]

is called a PAM (pulse amplitude modulated) signal. It can be transmitted directly (as in a baseband modem) or it can be used to modulate a carrier following one of the schemes discussed in the last chapter.

**Binary signaling**

Here \( L = 1 \) so there are only two symbols, -1 and +1 representing 1 and 0. Take the pulse shape to be a constant, \( p(t) = 1, 0 < t < T \). So if \( a(k) \in \{-1, +1\} \) is the \( k \)th symbol, the baseband signal during the \( k \)th symbol time is

\[
\forall t, \quad x(t) = a(k), \quad (k - 1)T \leq t < kT,
\]

as shown on the top in figure 4.2 for the case where \( \{a_k\} \) alternates between -1 and +1.

![Figure 4.2: The binary signal 1010···, the baseband signal \( x \), and the waveform produced using OOK, BPSK and FSK.](image)

**OOK** In *on-off* keying or OOK, the modulation scheme is AM, so the transmitted bandpass signal is

\[
\forall t, \quad y(t) = [1 + x(t)] \cos \omega_c t,
\]

as shown. OOK is used in optical communication: the laser is turned on for the duration of a bit time \( T \) to signal ‘1’ and turned off for the same amount of time to signal ‘0’.
BPSK In binary phase-shift keying or BPSK, phase modulation is used, so the transmitted bandpass signal is
\[ y(t) = \cos(\omega_c t + \beta x(t)). \]
In figure 4.2, \( \beta = \pi/2 \). So \( x(t) = 1 \) is transmitted as \( \cos(\omega_c t + \pi/2) = -\sin \omega_c t \) and \( x(t) = -1 \) is transmitted as \( \cos(\omega_c t - \pi/2) = \sin \omega_c t \). BPSK is used to transfer data over coaxial cable.

FSK In frequency shift keying or FSK, frequency modulation is used so the transmitted bandpass signal is
\[ y(t) = \cos(\omega_c + x(t)\omega_0) t. \]
So \( x(t) = 1 \) is transmitted as a sinusoid of frequency \( \omega_c + \omega_0 \), and \( x(t) = -1 \) is transmitted as a sinusoid of frequency \( \omega_0 - \omega_0 \).

Multilevel signaling

Each block of \( L \) bits is mapped into \( K = 2^L \) complex amplitudes, \( R_l e^{j\theta_l}, l = 1, \ldots, 2^L \), arranged symmetrically in the complex plane. The arrangement is called a constellation. The pulse shape is again a constant, \( p(t) = 1, 0 < t < T \). So if \( a(k) \in \{ R_l e^{j\theta_l} \} \) is the \( k \)th symbol, the complex baseband signal during the \( k \)th symbol time is
\[ x(t) = a(k), \quad (k - 1)T \leq t < kT. \]

QAM A 16-level quadrature amplitude modulation or QAM constellation is shown on the left in figure 4.3.

![Figure 4.3: 16-QAM constellation on the left, QPSK constellation on the right.](image)

The QAM modulated signal is
\[ y(t) = \Re\{x(t)e^{j\omega_c t}\} = x_c(t) \cos \omega_c t + x_s(t) \sin \omega_c t. \]
So the symbol \( x(t) = R_l e^{j \theta_l} \) is transmitted as
\[
y(t) = \text{Re}\{R_l e^{j(\omega_c t + \theta_l)}\} = R_l \cos \theta_l \cos \omega_c t - R_l \sin \theta_l \sin \omega_c t
\]
\[
= x_c(t) \cos \omega_c t + x_s(t) \sin \omega_c t.
\]
The waveform \( x_c \) is called the in-phase component and \( x_s \) is called the quadrature component.

Transmission of digital data downstream over a 6 MHz cable TV channel using 64 QAM can achieve a 28 Mbps bit rate, for a spectral efficiency of 10.76 bits/Hz. The symbol rate is \( 28/64 = 437.5 \) kilobaud/sec.

The QAM modulator is shown in figure 4.4. Note the resemblance to the AM-SSB modulator. The demodulator is similar.

**Figure 4.4:** The QAM modulator. LO is the local oscillator which produces the carrier signal.

**QPSK** Quadrature phase-shift keying or K-ary phase-shift keying is similar to QAM with \( K = 2^L \) levels, except that the amplitudes \( \{R_l e^{j \theta_l}\} \) have the same magnitudes \( R_l \equiv 1 \), say. Thus the information is contained in the phase. One possible 4-level QPSK constellation is shown in the right of figure 4.3.

**Spectral efficiency**

The bandpass signal \( y \) in the schemes above is centered around the carrier frequency \( \omega_c \). The bandwidth of the signal is the same as that of the baseband signal
\[
\text{for all } t, \quad x(t) = \sum a(k) p(t - kT),
\]
and it depends on the symbol sequence \( \{a(k)\} \) and the shape of the pulse \( p \).

Consider the case of binary signaling, with the symbol sequence alternating between +1 and -1, \( a(k) = (-1)^k \). Suppose the pulse is a squarewave as in figure 4.2. So the baseband signal is periodic with period \( 2T \):
\[
\forall t, \quad x(t) = (-1)^k, \quad (k - 1)T \leq t < kT.
\]
This periodic signal has a Fourier series, say,
\[
\forall t, \quad x(t) = \sum_{n=-\infty}^{\infty} P_n e^{j n\omega_0 t},
\]
and a FT $X$,

$$\forall \omega, \quad X(\omega) = 2\pi \sum_{-\infty}^{\infty} P_n \delta(\omega - n\omega_0).$$

Here $\omega_0 = 2\pi/T$. We can define the bandwidth of $x$ as follows. Let $N$ be the smallest number of harmonics that contain 95% of the power,

$$\frac{\sum_{-N}^{N} |P_n|^2}{\sum_{-\infty}^{\infty} |P_n|^2}.$$

The 95% bandwidth of the baseband signal $x$ is defined as $W_{95} = 2\pi \times 2N\omega_0$ Hz. This will be the bandwidth of the modulated signal.

Per unit time, this modulated signal carries $b = 1/T$ bits of information. So the spectral efficiency if this modulation scheme is $b/W_{95}$ bits/sec/Hz. The spectral efficiency depends on the shape of the pulse $p$. 