Chapter 13

Additional LT Exercises

Each problem is annotated with the letter E, T, C which stands for exercise, requires some thought, requires some conceptualization. Problems labeled E are usually mechanical, those labeled T require a plan of attack, those labeled C usually have more than one defensible answer.

1. E Determine the LT and RoC of the following signals: \( \forall t \in \text{Reals} \):
   
   (a) \( x(t) = u(t - 2) \)
   
   (b) \( x(t)e^{2t}u(-t + 2) \)
   
   (c) \( x(t) = \delta(t - t_0) \)
   
   (d) \( x(t) = \cos(2t)u(t) \)

2. E Determine the LT and RoC using LT tables and properties for the following signals: \( \forall t \in \text{Reals} \):
   
   (a) \( x(t) = t^2e^{-2t}u(t) \)
   
   (b) \( x(t) = e^{-t}u(t) \ast \sin(3\pi t)u(t) \)
   
   (c) \( x(t) = \frac{d}{dt}[t\delta(t)] \)
   
   (d) \( x(t) = tu(t) - (t - 1)u(t - 1) - (t - 2)u(t - 2) + (t - 3)u(t - 3) \)
   
   (e) \( x(t) = 2t\frac{d^2}{dt^2}(e^{-t}\sin(t)u(t)) \)

3. E Find the time signals of the following unilateral LT. \( \forall s \in \text{Roc} \):
   
   (a) \( \hat{X}(s) = \frac{1}{s(s+1)} \)
   
   (b) \( \hat{X}(s) = \frac{d}{ds}(e^{-2s}\frac{1}{(s+1)^2}) \)
   
   (c) \( \hat{X}(s) = \frac{1}{(s+2)^2+4} \)
   
   (d) \( \hat{X}(s) + s\frac{d^2}{ds^2}(\frac{4}{s^2+1}) \)

4. E The circuit of figure 13.1 has input voltage \( x \) and output voltage \( y \). Determine the response for the following conditions:
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Figure 13.1: Circuit for Problem 4

(a) \( R = 3\Omega, L = 1H, C = \frac{1}{2}F, \forall t, x(t) = e^{-3t}u(t)V \), current through inductor at \( t = 0^+ \) is 2A, voltage across capacitor at \( t = 0^+ \) is 1 V.

(b) \( R = 2\Omega, L = 1H, C = \frac{1}{5}F, \forall t, x(t) = 2e^{-t}u(t)V \), current through inductor at \( t = 0^+ \) is 2A, voltage across capacitor at \( t = 0^+ \) is 1 V.

5. E Determine the LT and corresponding RoC for the following signals: \( \forall t \in \text{Reals} \)
   
   (a) \( x(t) = e^{-2t}u(t) + e^t u(-t) \)
   
   (b) \( x(t) = e^{2t} \cos(2t)u(-t) + e^{-t}u(t) + e^t u(t) \)
   
   (c) \( x(t) = e^{-t} \frac{d}{dt}(e^{-t}u(t+1)) \)
   
   (d) \( x(t) = \int_{-\infty}^{t} e^{2\tau} \sin(\tau)u(-\tau)d\tau \)

6. E Use LT tables and properties to determine the inverse LT of the following LT: \( \forall s \in \text{RoC} \)
   
   (a) \( \hat{X}(s) = \frac{3}{s^2 + 5s - 1} \) with \( \text{RoC} = \{\text{Re}s > -2\} \)
   
   (b) \( \hat{X}(s) = \frac{2s^2 + 3}{s^2 + 2\sqrt{2}s + 4} \) with \( \text{RoC} = \{\text{Re}s > 0\} \)

7. E A signal \( x \) has LT \( \hat{X} \) given below. Plot the poles and zeros. Assume \( x \) is absolutely integrable, so \( \{\text{Re}s = 0\} \subset \text{RoC} \). Determine \( x \) and the FT of \( x \).
   
   (a) \( \hat{X}(s) = \frac{s^2 - 1}{s^2 + 3s + 2} \)
   
   (b) \( \hat{X}(s) = \frac{2s^2}{s^2 + 2\sqrt{2}s + 4} \)
   
   (c) \( \hat{X}(s) = \frac{1}{s - 3} + \frac{2}{s + 2} \)

8. E Determine \( x(0^+) \) for the following unilateral LT.
   
   (a) \( \hat{X}(s) = \frac{3}{s^2 + 5s - 1} \)
   
   (b) \( \hat{X}(s) = \frac{2s^2 + 3}{s^2 + 5s - 6} \)
   
   (c) \( \hat{X}(s) = e^{-s} \frac{3s^2 + 2s}{s^2 + s - 1} \)

9. E Determine the final value \( x(\infty) \) for the following unilateral LT.
   
   (a) \( \hat{X}(s) = \frac{2s^2 + 3}{s^2 + 5s + 1} \)
   
   (b) \( \hat{X}(s) = \frac{2s + 3}{s^2 + 5s^2 + 6s} \)
10. Use partial fraction expansion to find the time signal corresponding to the following unilateral LT.

(a) \( \hat{X}(s) = \frac{-s-4}{s^2+3s+2} \)

(b) \( \hat{X}(s) = \frac{6}{s^2+6s+6} \)

(c) \( \hat{X}(s) = \frac{2s-1}{s^2+2s+1} \)

(d) \( \hat{X}(s) = \frac{5s+4}{s^3+3s^2+2s} \)

(e) \( \hat{X}(s) = \frac{3s^2+8s+5}{(s+2)(s^2+s+1)} \)

(f) \( \hat{X}(s) = \frac{3s+2}{s^2+4s+5} \)

(g) \( \hat{X}(s) = \frac{4s^2+8s+10}{(s+2)(s^2+2s+5)} \)

(h) \( \hat{X}(s) = \frac{-9}{(s+1)(s^2+2s+10)} \)

(i) \( \hat{X}(s) = \frac{s+4+e^{-s}}{s^2+5s+6} \)

11. Determine the response \( y \) for two cases, zero initial condition and zero input.

(a) \( 5\ddot{y} + 10y = 2z, \ y(0+) = 2, \forall t, x(t) = u(t) \)

(b) \( \ddot{y} + 5\dot{y} + 6y = -4x - 3\dot{x}, \ y(0+) = -1, \ \dot{y}(0+) = 5, \forall t, x(t) = e^{-t}u(t) \)

(c) \( \ddot{y} + 4y = 8x, \ y(0+) = 1, \ \dot{y}(0+) = 2, \forall t, x(t) = u(t) \)

(d) \( \ddot{y} + 2\dot{y} + 5y = \dot{x}, \ y(0+) = 2, \ \dot{y}(0+) = 0, \forall t, x(t) = e^{-t}u(t). \)

12. Consider the circuit of figure 13.2.

(a) Define the state vector \( x(t) = (i(t), v(t))' \). Write down the differential equation for \( x \).
(b) Define \( x_k = x(kT) \), the sampled value of \( x \). Obtain a difference equation of the form

\[
x_{k+1} = Ax_k + Bu_k,
\]

by using the approximation \( \dot{x}(t)|_{t=kT} = \frac{1}{T}(x_k - x_{k-1}) \).

13. E Use LT to find the transfer function and impulse response.
   
   (a) \( 5\ddot{y} + 10y = 2x \)
   (b) \( \ddot{y} + 5\dot{y} + 6y = x + \dot{x} \)
   (c) \( \ddot{y} - 2\dot{y} + 10y = x + 2\dot{x} \)

14. E Find a differential equation whose transfer function is the following.
   
   (a) \( \hat{H}(s) = \frac{2s+1}{s(s+2)} \)
   (b) \( \hat{H}(s) = \frac{3s}{s^2+2s+10} \)
   (c) \( \hat{H}(s) = \frac{2(s+1)(s-2)}{(s+1)(s+2)(s+3)} \)

15. T Suppose a system has \( M \) poles at \( d_k = \alpha_k + j\beta_k \) and \( M \) zeros at \( c_k = -\alpha_k + j\beta_k \), that is, the poles and zeros are symmetric about the \( j\omega \)-axis.
   
   (a) Show that the magnitude response is identically 1. Such a system is called allpass.
   (b) Find the phase response of a single pole-zero pair, i.e. the phase response of \( (s-\alpha)/(s+\alpha) \).

16. E Sketch the phase response of the following transfer functions.
   
   (a) \( \hat{H}(s) = \frac{s-1}{s+2} \)
   (b) \( \hat{H}(s) = \frac{s+1}{s+2} \)
   (c) \( \hat{H}(s) = \frac{1}{s^2+2s+10} \)
   (d) \( \hat{H}(s) = s^2 \)

17. E Use the matlab function \texttt{roots} to determine the poles and zeros of the following.
   
   (a) \( \hat{H}(s) = \frac{s^2+2}{s^3+2s^2-s+1} \)
   (b) \( \hat{H}(s) = \frac{s^3+1}{s^4+2s^2+1} \)
   (c) \( \hat{H}(s) = \frac{4s^2+8s+10}{2s^3+8s^2+18s+20} \)

18. T Use the matlab function \texttt{freqresp} to plot the magnitude and phase response for problems 13, 14.

19. T Consider the LTI system with transfer function \( H(s) = 1/(s + 2) \) and suppose we want to modify the system by adding negative feedback such that the closed-loop transfer function is \( G(s) = (s + 1)/(s^2 + 3s + 3) \).
   
   (a) What is the transfer function of the compensator?
(b) Sketch the Bode plots of the open loop, compensator, and closed loop.

20. **E** A stable LTI system has frequency response

\[ H(\omega) = \begin{cases} 
  j, & 0 \leq \omega \leq \pi \\
  -j, & -\pi \leq \omega \leq 0 \\
  0, & \text{otherwise}
\end{cases} \]

(a) What is the impulse response of the system?
(b) Is it causal?
(c) Does the system give a real output for all real inputs?

21. **E** The step response of an LTI system is given by

\[ s(t) = \begin{cases} 
  0, & t \leq 0 \\
  1 - 0.5e^{-t} + 0.5e^{-2t}, & t > 0
\end{cases} \]

(a) Find its impulse response, transfer function, and frequency response.
(b) Find its steady state response to the input

\[ \forall t, \quad x(t) = \cos(\omega t)u(t). \]

(c) Find its response to the input

\[ \forall t, \quad x(t) = \cos(\omega t). \]

22. **E** Determine which of the following transfer functions are stable and for those that are, determine the initial and steady state values to a step input.

\[
\frac{s - 1}{s^4 + 1}, \quad \frac{s^2 - s + 1}{s^4 + 3s^2 + 1}, \quad \frac{s + 1}{s^4 + 2s^3 + 3s^2 + 4s + 1}
\]

23. **T** The open loop plant has transfer function \( H(s) = \frac{s^2}{s^2 + 2s + 1} \). Place the plant in a closed loop using a PI controller \( K_1 + K_2/s \).

(a) Take \( K_2 = 0 \), and plot the root locus as \( K_1 \) varies. For what values of \( K_1 \) is the system stable? What is the steady state error to a step input as a function of \( K_1 \)?
(b) Select \( K_1, K_2 \) such that the closed loop system is stable and has zero steady-state error for step inputs.