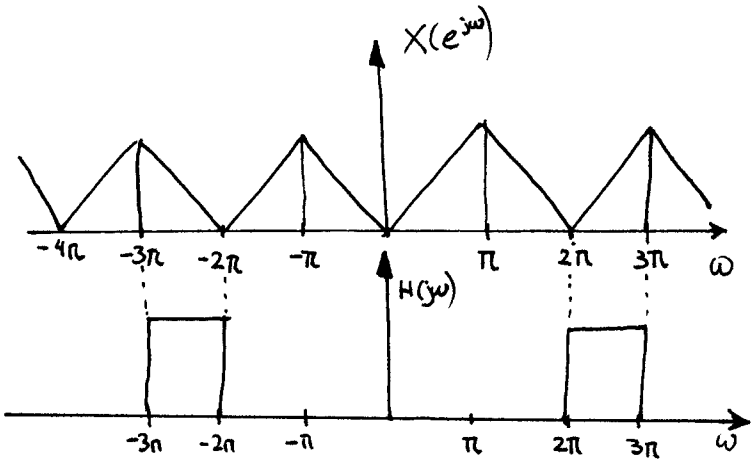
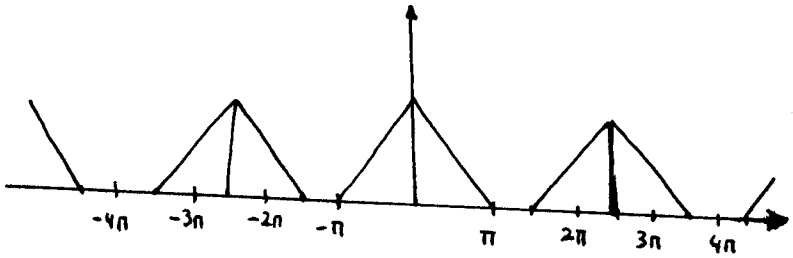
Homework 6

Problem 1

For $\omega_s = 2\pi$:





Nyquist frequency: 5π

But we can sample at $\omega_s = \frac{5}{2}\pi$

(Which is the smallest rate that avoids aliasing)

We want the aliased copies to be non-overlapping

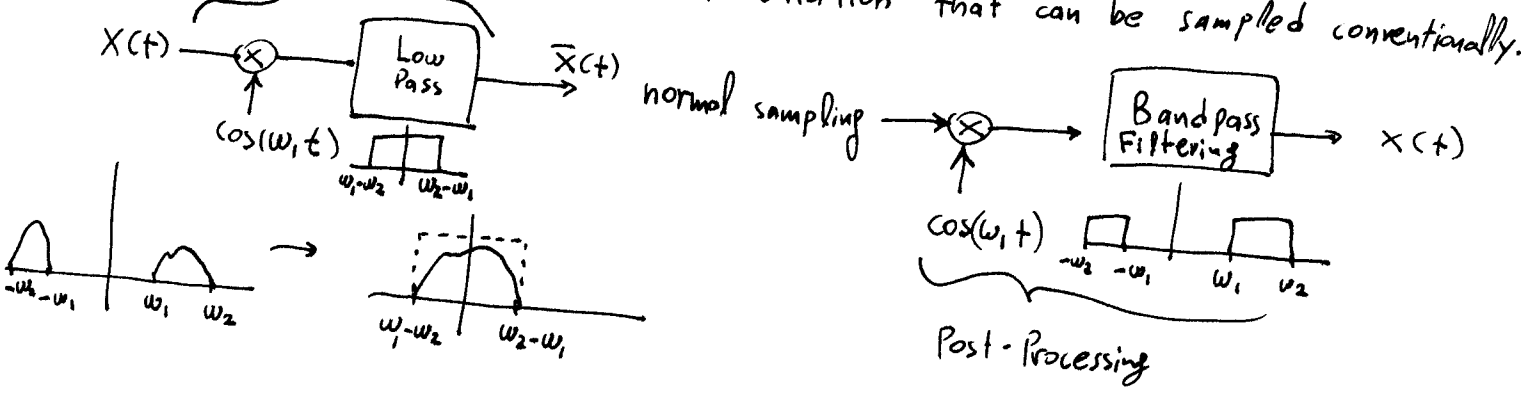
$\Rightarrow$ ω_1 must be a multiple of the bandwidth:

$$\omega_1 = k\,(\omega_2 - \omega_1) \quad \text{where } k \text{ integer.}$$

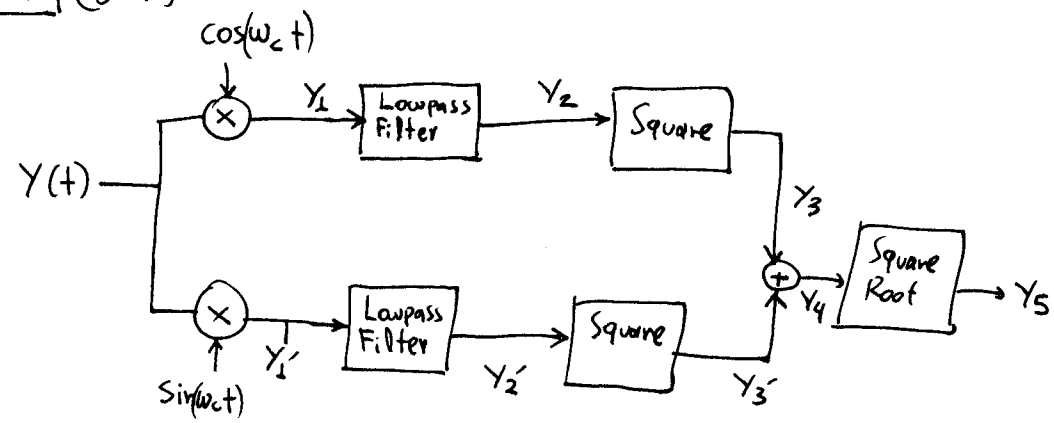
Therefore the lowest sampling frequency is

$$\omega_s = \frac{2\omega_2}{c} \quad \text{where} \quad c = \left\lfloor \frac{\omega_2}{\omega_2 - \omega_1} \right\rfloor = \text{number of complete copies of the spectrum that can fit in } |\omega| < \omega_2$$

Preprocessing: We can always shift the bandpass signal and make a lowpass representation that can be sampled conventionally.



Prob 3 | (8.26)



$$Y(t) = [x(t) + A]\cos(\omega_c t + \vartheta_c)$$

$$Y_1(t) = (x(t)+A)\cos(\omega_c t + \vartheta_c)\cdot\cos(\omega_c t) = [x(t)+A]\left[\frac{1}{2}\cos[\vartheta_c] + \frac{1}{2}\cos[2\omega_c t + \vartheta_c]\right]$$

$$Y_2(t) = [x(t) + A]\left[\frac{1}{2}\cos(\vartheta_c)\right]$$

$$Y_3(t) = (x(t) + A)^2 \; \frac{1}{4}\cos^2(\vartheta_c).$$

And

$$Y_1'(t) = (x(t)+A)\cos(\omega_c t + \vartheta_c)\cdot\sin(\omega_c t) = (x(t)+A)\left[\frac{1}{2}\sin(2\omega_c t + \vartheta_c) - \frac{1}{2}\sin(\vartheta_c)\right]$$

$$Y_2'(t) = (x(t) + A)\left(-\frac{1}{2}\sin\vartheta_c\right)$$

$$Y_3'(t) = (x(t) + A)^2 \; \frac{1}{4}\sin^2\vartheta_c$$

Therefore

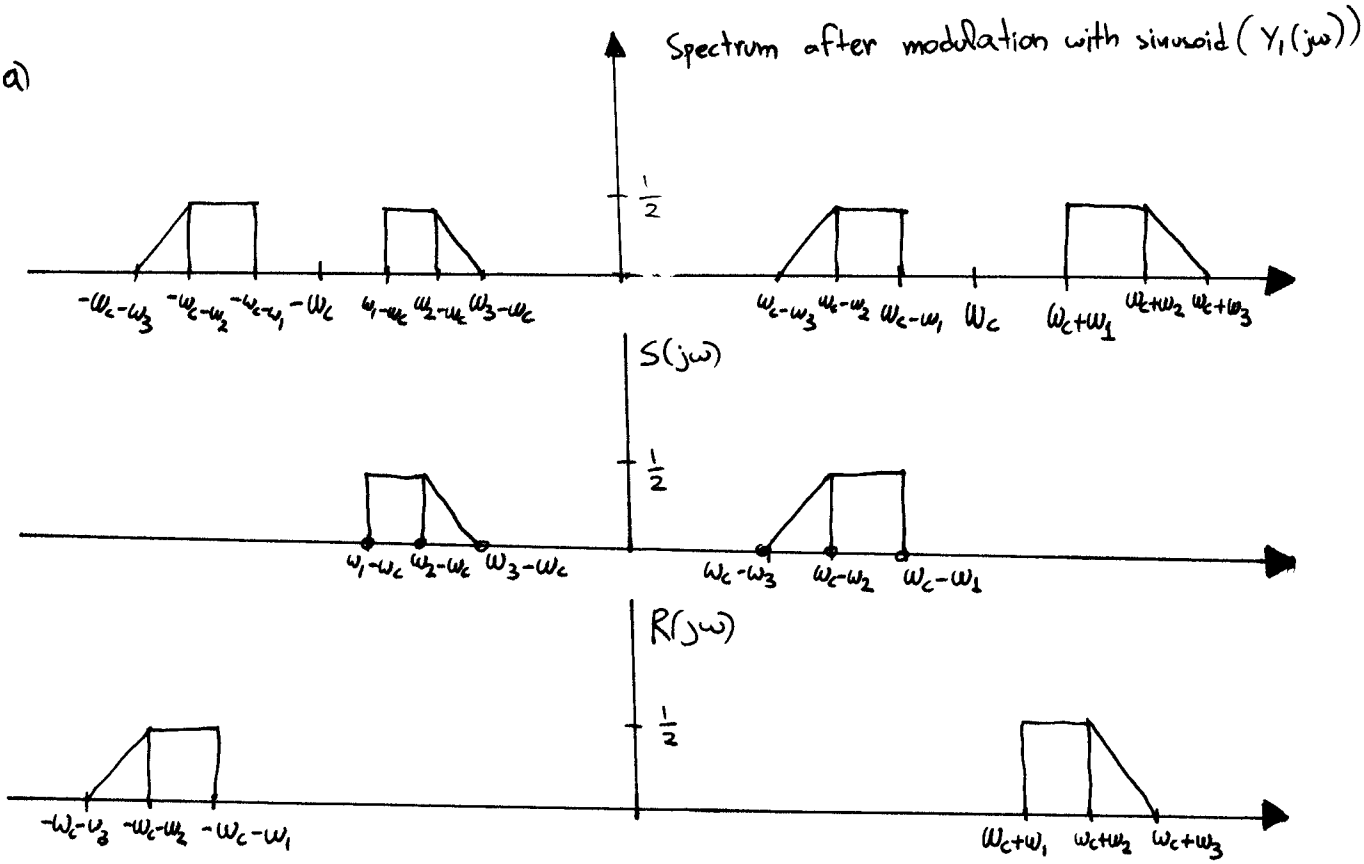
$$Y_5(t) = \sqrt{(x(t)+A)^2 \; \frac{1}{4}\cos^2(\vartheta_c) + (x(t)+A)^2\cdot\frac{1}{4}\sin^2(\vartheta_c)} = |x(t)+A|\sqrt{\frac{1}{4}(\cos^2\vartheta_c + \sin^2\vartheta_c)}$$

$$= \frac{1}{2}|x(t)+A| = \frac{1}{2}(x(t)+A) \quad \text{since } x(t)+A \text{ is positive}$$

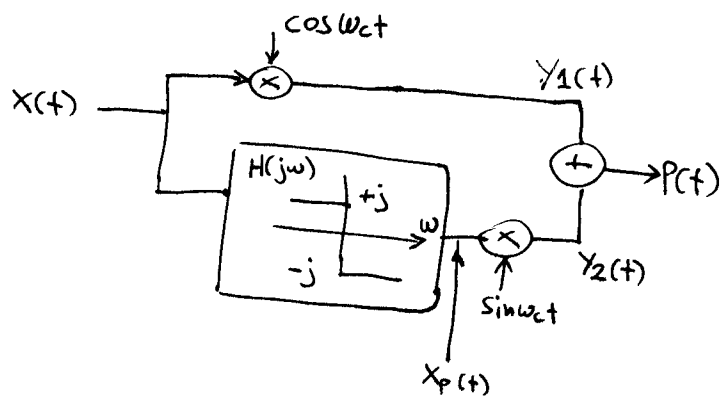
Therefore we recover $x(t)$ by rescaling and subtracting A.

Prob 4) 8.29

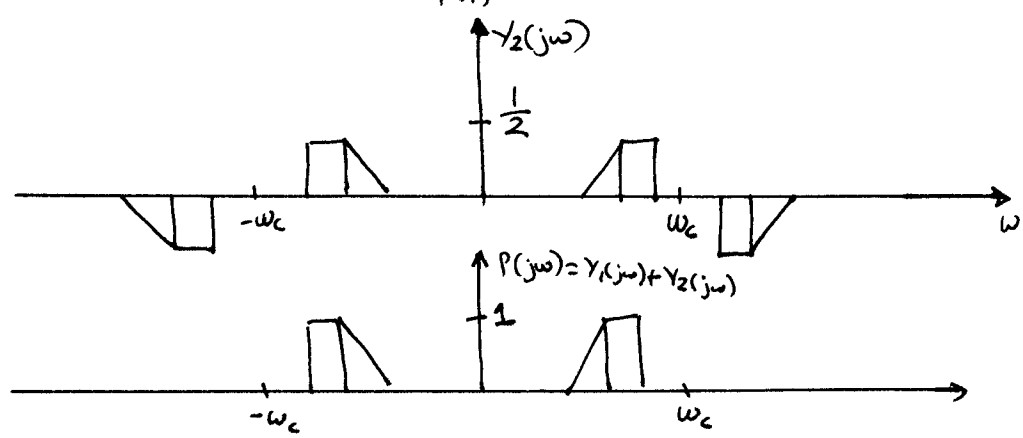
a)



b).

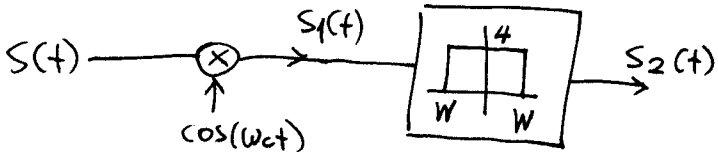


The spectrum of Y_2 will be:



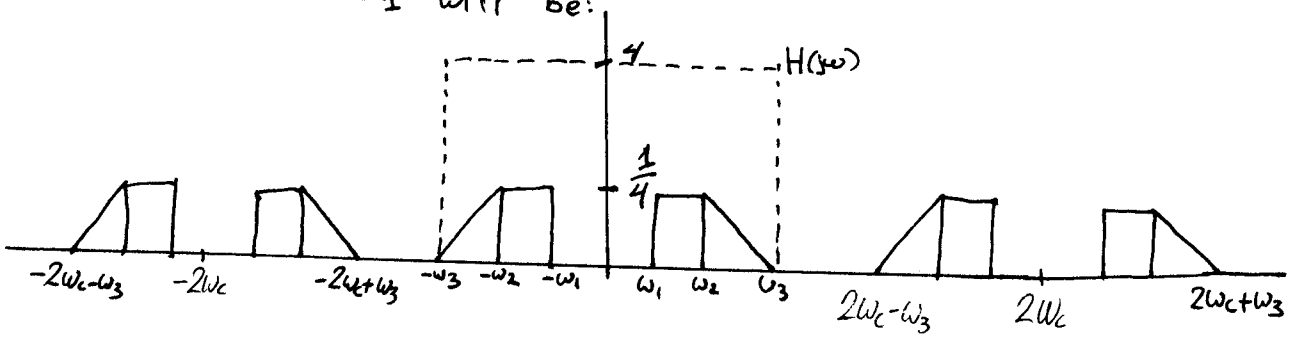
Notice that $P(j\omega) = 2\,S(j\omega)$ so they are propotional.
(Since the upper side band is canceled out by its opposite).

Prob 4) c)



We want to show that $S_2(t) = X(t)$.

The spectrum of S_1 will be:



And therefore the filtered signal $S_2(j\omega) = X(j\omega)$