EECS 120 Signals & Systems — University of California, Berkeley: Spring 2004
Gastpar — March 17, 2004

Homework 7
Due: Thursday, April 1, 2004, at 4pm

Reading OWN Chapter 8.

Practice Problems *(Suggestions.)* OWN 8.8, 8.13, 8.20.

Problem 1 *(PAM with fading.)* — 25 Points

Consider a PAM system as discussed in class: A discrete-time data signal $s[n]$ is used to produce the following modulated signal:

$$x(t) \quad = \quad \sum_{n=-\infty}^{\infty} s[n]p(t-nT), \tag{1}$$

where $p(t)$ is a box function that starts at time zero and ends at time $\Delta < T$, of height one.

The signal $x(t)$ is passed through a fading communication channel with frequency response

$$H(j\omega) \quad = \quad \frac{1}{a+j\omega}, \tag{2}$$

where a is *positive and real-valued.* Call the channel output signal $y(t)$.

(a) For $a = T$ and

$$s[n] \quad = \quad \begin{cases} 1, & n = 0, 2 \\ 2, & n = 1, 3, \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

sketch the signal $y(t)$ at the receiver. Carefully label the time axis. *Hint:* Consider a single pulse first, and use linearity.

(b) The channel output signal $y(t)$ is sampled at times nT to yield the signal $y[n]$. Write $y[n]$ in terms of $s[n]$ (for general $s[n]$, not merely for the $s[n]$ of Part *(a)*). Can the relationship between $s[n]$ and $y[n]$ be understood as a discrete-time linear time-invariant (LTI) system? If so, determine its impulse response $h[n]$.

(c) Find a discrete-time LTI system with impulse response $g[n]$ and frequency response $G(e^{j\omega})$ that takes the received signal $y[n]$ and outputs the original transmitted signal $s[n]$.

(d) Generally, at the receiver, you do not exactly know the constant a that governs the channel behavior, but you can estimate it, giving you some $\hat{a}$ which is hopefully close to the true a. Moreover, since the decoder that you designed in Part *(c)* will operate in digital logic, you have to round $\hat{a}$ to the closest number $\tilde{a}$ that can be represented on your DSP chip. But if the decoder of Part *(c)* uses $\tilde{a}$ instead of a, it will recover a signal $\tilde{s}[n]$ that is not exactly equal to $s[n]$. Find the error, defined as

$$E \quad = \quad \sum_{n=-\infty}^{\infty} |s[n] - \tilde{s}[n]|^2, \tag{4}$$

as a function of the difference $(\tilde{a} - a)$ (that is, your estimation error).

Problem 2 *(Super-heterodyne receivers.)* 25 Points

OWN 8.36.

Problem 3 *(Simple frequency shift keying communication system.)* 25 Points

OWN 8.39.

Problem 4 *(FM.)* 25 Points

OWN 8.45.