Homework 9
Due: Thursday, April 22, 2004, at 4pm

Reading OWN Chapter 9: 9.4 (skim 9.4.2, 9.4.3), 9.7.3, 9.7.4, 9.7.5, 9.8.1; 6.2.3 (Bode plots).

Practice Problems *(Suggestions.)* OWN 9.10, 9.11, 9.12, 9.15, 9.17.

Problem 1 *(Pole/Zero Plots)* 10 Points

Match the pole/zero plots (a)-(e) with the corresponding magnitude responses (1)-(5). In each case, provide a brief justification. (For example: "must have two symmetric peaks, therefore can only be plot (x)".)

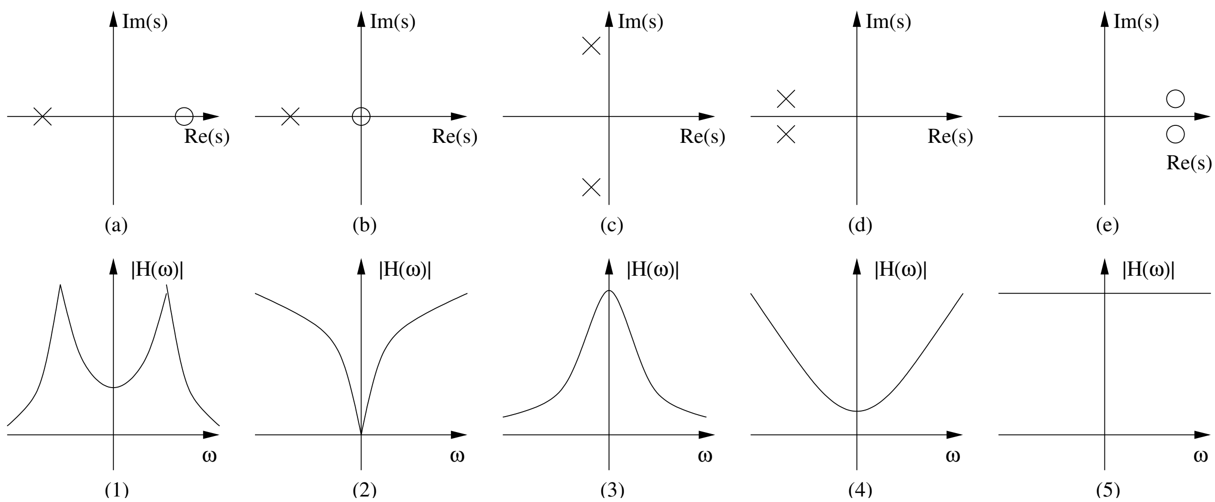

Figure 1: Matching of Pole/Zero Plots and Frequency response.

Problem 2 *(A simple feedback control system)* 20 Points

One of the key applications of the Laplace transform is in the control of feedback systems. Consider the following simple feedback system.

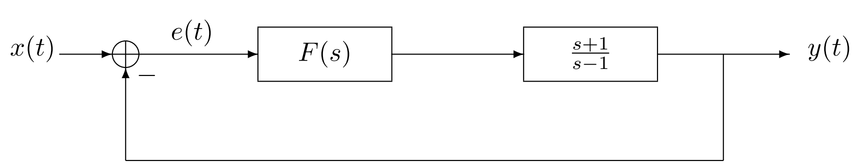

Figure 2: A simple feedback system

The second box, $\frac{s+1}{s-1}$, models an industrial plant, and the task of the engineer is to design the "compensator" $F(s)$ in a clever way. The overall goal of the control is to make the error signal $e(t) = 0$.

The system satisfies

$$Y(s) = \frac{s+1}{s-1} F(s) \ (X(s) - Y(s)), \tag{1}$$

and hence, the overall transfer function is

$$T(s) = \frac{Y(s)}{X(s)} = \frac{\frac{s+1}{s-1} F(s)}{1 + \frac{s+1}{s-1} F(s)}. \tag{2}$$

Moreover, observe that the Laplace transform of the error signal satisfies

$$E(s) = (1 - T(s)) X(s). \tag{3}$$

(a) Suppose that $F(s) = K$, where K is a real number. Determine the range of K such that the overall system $T(s)$ is stable. Assume that K is chosen such that the overall system is stable. For $x(t) = u(t)$, what is the asymptotic value of the error $e(t)$ as $t \to \infty$? For $x(t) = e^t u(t)$, what is the asymptotic value of the error $e(t)$ as $t \to \infty$?

(b) Repeat *(a)* assuming that the compensator also contains an integrator, hence $F(s) = K/s$, where K is a real number.

Problem 3 *(Bode Plots)* 25 Points

As we have seen in class, the Bode plot of a frequency response is simply to plot

$$|H(j\omega)|_{dB} \stackrel{def}{=} 20 \log_{10} |H(j\omega)|. \tag{4}$$

(a) (5 Points) In this problem, we use matlab to confirm Bode's approximation. Consider the system with transfer function

$$H(s) = \frac{1}{1 + s/10}. \tag{5}$$

Use the following matlab code:

```
w = [ 0.1:0.1:1000 ];
semilogx(w, 20*log10(abs(1./(1 + j*w/10))));
grid;
```

Print out the resulting plot, and add (with a color pen) the approximation that we have seen in class. Then, repeat this exercise for the phase, using the following matlab code:

```
w = [ 0.1:0.1:1000 ];
semilogx(w, angle(1./(1 + j*w/10)));
grid;
```

(b) (10 Points) Repeat question *(a)* for the second-order system

$$H(s) = \frac{1}{1 + s/20 + (s/10)^2}. \tag{6}$$

For the *phase response*, derive an approximation for very small ω and for very large ω, along the lines of what we did in class (and in Part *(a)* of this problem).

(c) (10 Points) By hand (without using matlab), provide the Bode plot of the *magnitude* of the frequency response of the system with transfer function

$$H(s) = \frac{(s+10)(s+100)}{10(s+1)(s+1000)}. \tag{7}$$

Describe the system behavior in words, and determine the differential equation that describes this system.

Hint: Confirm your result using matlab, but be sure you know how to do it by hand.

Problem 4 *(Laplace methods)* 20 Points

(a) OWN Problem 9.35 (a), (b)

(b) OWN Problem 9.47 (a), (b). Moreover, is this system a minimum phase system?

Problem 5 *(Eigenfunctions.)* 25 Points

A signal (or function) $x_0(t)$ is called an *eigenfunction* of a system H if it is true that

$$H\{x_0(t)\} \quad = \quad \lambda_0 x_0(t), \tag{8}$$

for some constant λ_0. The constant λ_0 is called the eigenvalue corresponding to the eigenfunction $x_0(t)$. Generally, a system has *many different* eigenfunctions with their corresponding eigenvalues. In words, if the input is an eigenfunction, the output is just a scaled version of the input signal!

(a) (4 Points) We have seen earlier that the complex exponentials $e^{j\omega t}$ are eigenfunctions of LTI systems. Consider the signal

$$x(t) \quad = \quad e^{j\omega_0 t}, \tag{9}$$

where ω_0 is a real number. Determine the eigenvalue corresponding to the signal $x(t)$ using the frequency response $H(j\omega)$ of the LTI system. *Hint:* If you are unsure, read OWN, Section 3.2, again. Your answer is very simple and depends on ω_0.

(b) (7 Points) Prove that the functions

$$x(t) \quad = \quad e^{s_0 t}, \tag{10}$$

where s_0 is a *complex-valued* constant, are *also* eigenfunctions of LTI systems, and determine the corresponding eigenvalues as a function of the parameter s_0.

(c) (7 Points) OWN Problem 9.32 *Remark.* The constant b can be determined from the information given in the problem statement.

(d) (7 Points) Let us now consider more general systems that are *not* LTI. Instead, suppose that the system H is known to be linear and time-variant, and characterized by the following relationship:

$$y(t) = H\{x(t)\} \quad = \quad t^3 x'''(t), \tag{11}$$

where $x'''(t)$ denotes the third derivative of $x(t)$ with respect to t. Show that the functions $\psi_k(t) = t^k$, $k \in R$, are eigenfunctions of this system, and find their eigenvalues.

Problem 6 *(Unilateral Laplace transform)* Extra Credit 10 Points

Remark. The unilateral Laplace transform is not part of this course and will not be part of the final exam.

Read Section 9.9 of the course textbook and solve Problem 9.40(b). "Zero input" means that $x(t) = 0$ for all t.