Homework 5
Due: Thursday, March 3, 2005, at 11:30am

Reading OWN Chapters 4, 5, 7.

Please write your section day and time on the upper left of the front page of your homework. This will help us return your homeworks.

You may work in (small) groups to do the homework, but each person must write up their own answers. Note that working together does not mean dividing up the problems and sharing answers later.

For any Matlab problems, submit computer generated plots only. For any question requiring a plot, feel free to use Matlab. **No code is required!**

Problem 1 *(Frequency response of linear time-invariant system.)*

Let a system be specified by a differential (or difference) equation. Then its frequency response can be found easily, as you establish in this homework problem.

(a) In the continuous-time case, suppose that the LTI system is specified by

$$\sum_{k=0}^{N} a_k \frac{d^k}{dt^k} y(t) \quad = \quad \sum_{m=0}^{M} b_m \frac{d^m}{dt^m} x(t).$$

Solve the differential equation for the input $x(t) = e^{j\omega t}$. As we have seen in class, if the input to a continuous-time LTI system is $x(t) = e^{j\omega t}$, then the output can be expressed as $y(t) = H(j\omega)e^{j\omega t}$. Plug this into the above differential equation to determine $H(j\omega)$. You will obtain an expression for $H(j\omega)$ in terms of the coefficients $a_k, k = 0, \ldots, N$ and $b_m, m = 0, \ldots, M$.

(b) In the discrete-time case, suppose that the LTI system is specified by

$$\sum_{k=0}^{N} a_k y[n-k] \quad = \quad \sum_{m=0}^{M} b_m x[n-m].$$

Solve the differential equation for the input $x[n] = e^{j\omega n}$. As we have seen in class, if the input to a discrete-time LTI system is $x[n] = e^{j\omega n}$, then the output can be expressed as $y[n] = H(e^{j\omega})e^{j\omega n}$. Plug this into the above differential equation to determine $H(e^{j\omega})$. You will obtain an expression for $H(e^{j\omega})$ in terms of the coefficients $a_k, k = 0, \ldots, N$ and $b_m, m = 0, \ldots, M$.

(c) In previous homeworks, you solved simple differential equation. For the system specified by the differential equation, i.e.,

$$5\frac{dy(t)}{dt} + 10y(t) \quad = \quad x(t),$$

determine the frequency response $H(j\omega)$, and the corresponding impulse response $h(t)$. Then, find the output when the input is $x(t) = \cos(t/5)$.

OWN 4.7 *(CTFT properties)*

Answers are in the back of the book, so justify your answers. However, please be brief; rigorous proofs are unnecessary.

OWN 4.21 *(CTFT)*

Parts a, b, c, j

OWN 4.23 *(CTFT properties)*

OWN 4.25 *(CTFT properties)*

Parts b and e

OWN 4.36 *(CTFT and differential equations)*

OWN 5.21 *(DTFT)*

Parts b, g, i, k

OWN 5.22 *(DTFT)*

Parts c, e, f

If we do not cover sampling in class before this homework is due, the next two questions will be postponed to the following week's homework.

OWN 7.21 *(Sampling)*

You do not need to justify your answers for this question, but no partial credit will be given if you don't. (In other words, it's all or nothing unless you give an explanation for your answers.)

OWN 7.22 *(Sampling)*