Homework 7
Due: Thursday, March 31, 2005, at 11:30am

Reading OWN Chapters 7,8.

Please write your section day and time on the upper left of the front page of your homework. This will help us return your homeworks.

You may work in (small) groups to do the homework, but each person must write up their own answers. Note that working together does not mean dividing up the problems and sharing answers later.

For any Matlab problems, submit computer generated plots only. **No code is required!**

Problem 1 *(Downsampling and Upsampling.)*

(a) OWN 7.35

(b) OWN 7.47

(c) OWN 7.49

Problem 2 *(DT Zero Order Hold.)*

OWN 7.50

Problem 3 *(Duality and Frequency Domain Sampling.)*

OWN 7.52

Problem 4 *(Amplitude Modulation.)*

(a) OWN 8.21

(b) OWN 8.22

Problem 5 *(MATLAB problem on upsampling/downsampling.)*

Parts of this problem are similar to the matlab problem from last week. You may use my code from last week's solutions if you desire.

First, note that the signal

$$\begin{aligned} x(t) &= \left(\frac{W}{4\pi}\right)^2 \operatorname{sinc}^2\left(\frac{Wt}{4\pi}\right) \\ &= \left(\frac{\sin(\frac{W}{4}t)}{\pi t}\right)^2 \end{aligned}$$

has Fourier transform

$$X(j\omega) = \begin{cases} 1 - \left|\frac{2\omega}{W}\right| & \text{if } -\frac{W}{2} < \omega < \frac{W}{2} \\ 0 & \text{elsewhere} \end{cases}$$

which is bandlimited with one sided bandwidth $\omega_M = \frac{W}{2}$. From the sampling theorem we know that if the signal is sampled with sampling interval T satisfying $\frac{2\pi}{T} > W$, then we can reconstruct $x(t)$ perfectly.

For this problem, pick $W = 8$. If the sampling interval is $T = 0.5$ we would have perfect reconstruction, while with $T = 2$ we should expect to see aliasing.

Using Matlab:

(a) Plot $x(t)$ over $-10 \leq t \leq 10$. Also plot $X(j\omega)$.

(b) For $T = 0.1$, plot $x_p(t)$, the result of multiplying $x(t)$ by an impulse train. For simplicity, since we must make approximations to account for Matlab's finite precision and discrete-time-ness, just make your impulses height 1. Be sure your axes are labeled correctly. Plot $X_p(j\omega)$. What is its period?

(c) For $T = 0.1$, plot $x_d[n]$, the result of converting $x_p(t)$ to discrete time. Be sure your axes are labeled correctly. Plot its Fourier transform. What is its period?

(d) Note that we're sampling well above the Nyquist frequency. We can therefore downsample the signal (to save memory, for faster processing, etc).

Let $y_1[n] = x_d[5n]$. Plot $y_1[n]$ and its Fourier transform.

(e) Assume our machinery for converting back to the analog domain operates at the same sampling period as our analog-to-digital converter. This means that we will have to upsample y_1 in order to convert it back to analog.

Let $y_2[n] = \begin{cases} y_1[n/5] & \text{if } n \text{ is divisible by } 5 \\ 0 & \text{otherwise} \end{cases}$

Plot $y_2[n]$ and its Fourier transform.

(f) Convert $y_2[n]$ back to continuous time by weighting an impulse train of period $T = 0.1$ by the coefficients of $y_2[n]$ and then low pass filtering the result. You may use whatever low pass filter you want. Turn in a plot of the resulting signal and a couple sentences describing your method of low pass filtering it.

Do NOT submit any M-files.