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Homework 9  
Due: Friday, April 22, 2005, at 5pm

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**Reading** OWN Chapter 9.

**Please write your section day and time on the upper left of the front page of your homework.** This will help us return your homeworks.

You may work in (small) groups to do the homework, but each person must write up their own answers. Note that working together does not mean dividing up the problems and sharing answers later.

For any Matlab problems, submit computer generated plots only. **No code is required!**

**Problem 0** (*Study for Quiz 2 - it is on Wednesday, April 20th.*)

**Problem 1** (*Region of Convergence.*) OWN 9.23, all parts.

**Problem 2** (*An LTI system.*)

For a linear time-invariant system, it is known that the system function (also called *transfer function*) is given by

$$H(s) = \frac{5(s-3)}{(s+2)(s^2-4s+13)}.$$

(a) Draw the pole/zero diagram for  $H(s)$ .

(b) Suppose that apart from  $H(s)$ , you are also told that the system is *causal*. Find the corresponding impulse response  $h(t)$  of the system. Is the resulting system also stable?

(c) Suppose that apart from  $H(s)$ , you are also told that the system is *stable*. Find the corresponding impulse response  $h(t)$  of the system. Is the resulting system also causal?

**Problem 3** (*Stability and Causality.*)

OWN 9.50

**Problem 4** (*LTI Systems modeled by Differential Equations.*)

OWN 9.31

**Problem 5** (*Echos in a Telecommunication System.*)

OWN 9.60

**Problem 6** (*MATLAB - Bode Diagrams.*)

In this problem, we will use Matlab to draw *exact* Bode Diagrams of transfer functions and compare them to the straight line approximations. See 'Handout 3' on the website for a few approximations.

(a) Consider the transfer function

$$H(s) = \frac{1}{1 + \frac{s}{\alpha}}$$

Let's look at  $\alpha = 1$ .

Draw the Bode Diagram of  $H(s)$ , using the straight line approximation developed in class. If you don't remember it, quickly rederive it for yourself.

Download `polebode.m` from the website. Go through the code and read the help on any functions you don't know. Run the code and look at the Bode diagram, checking it with your hand-drawn approximation. Do not turn in the plot given by this code.

What is the maximum difference between the approximation and exact Bode diagrams, both in phase (degrees) and magnitude (decibels)? At what frequencies do these maximum differences occur? You will find the MATLAB function '`max`' of use.

(b) Now let's look at the case of complex conjugate *zeros* in the left half plane.

$$H(s) = 1 + \frac{2\gamma}{\omega_n}s + \frac{s^2}{\omega_n^2}, \quad \omega_n > 0 \quad \text{and} \quad 0 \leq \gamma \leq 1$$

This transfer function has zeros at  $-\gamma\omega_n \pm j\omega_n\sqrt{1-\gamma^2}$

Let's look at the case for  $\gamma = \frac{1}{2}$  and  $\omega_n = 1$ .

$$H_1(s) = 1 + s + s^2$$

$H_1(s)$  has zeros at  $-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$ .

Develop a straight-line approximation for the magnitude and phase for  $H_1(s)$ . Draw your approximation by hand. Now, plot the Bode Diagram using the exact values and your approximation as in part a. Feel free to use the code from the last part.

(Note: You need to take into account both  $\gamma$  and  $\omega_n$  to get a good approximation of the phase. Don't worry about  $\gamma$  in approximating the magnitude.)

Turn in the plot of the Bode Diagram, and indicate the values and location of the maximum errors on the plot.

(c) Draw, by hand, the approximate Bode Diagram of the following transfer function.

$$H(s) = \frac{1 + 2s + s^2}{(1 + 10s)(1 + \frac{s}{100})} = \frac{(1 + s)(1 + s)}{(1 + 10s)(1 + \frac{s}{100})}$$

Check your hand drawn diagram with MATLAB's '`bode`' command.