

# EE 121: Introduction to Digital Communication Systems

Problem Set for Review Discussion Section

Thu 5/15/2008

## 1. Passband Communications

Let  $x_b(t)$  be the complex baseband representation of  $x(t)$ . Assume  $x_b(t)$  is bandlimited to  $(-W/2, W/2)$ . The signal  $s(t)$  is transmitted through an LTI channel with impulse response  $h(t)$ .

- (a) Draw and label a system diagram illustrating the upconversion from inphase and quadrature transmitted signals,  $x_I(t)$  and  $x_Q(t)$ , to  $x(t)$ , the passing of  $x(t)$  through the channel  $h(t)$ , and the subsequent downconversion of the received signal  $y(t)$  back to into inphase and quadrature received signals,  $y_I(t)$  and  $y_Q(t)$ .
- (b) Compute an expression for the FT  $X(f)$  of  $x(t)$  in terms of  $X_b(f)$ .
- (c) Compute an expression for  $y_I(t)$  and  $y_Q(t)$  in terms of  $x_I(t)$  and  $x_Q(t)$ .

## 2. Maximum Likelihood Sequence Detection (MLSD) for Convolution Codes

Consider the following error correction code. An information sequence  $\{b[n]\}$  consisting of i.i.d. equiprobable binary symbols 0 and 1, is transformed into a binary coded sequence  $\{x[n]\}$  as follows

$$x[2n] = b[n] \oplus b[n - 2] \quad (1)$$

$$x[2n + 1] = b[n] \oplus b[n - 1] \quad (2)$$

for  $n = 0, 1, \dots$ , where  $\oplus$  is addition mod 2. The coded sequence is mapped onto the channel such that the real received sequence is

$$y[n] = \sqrt{E}(2x[n] - 1) + w[n]$$

where  $w[n]$  are i.i.d Gaussian random variables with zero mean and variance  $\sigma^2$ .

- (a) An example: for  $\{b[n]\} = \{0, 1, 1, 0, 0\}$ , what is  $\{x[n]\}$ ? What is the length of the coded sequence?
- (b) What is the rate of the code?
- (c) What is the state vector? How many states are there? Draw a corresponding trellis diagram.
- (d) Suppose  $E = 1$ . Use MLSD to decode  $b[0], \dots, b[4]$  when the received sequence is  $\{y[n]\} = \{0.5, -0.5, 1.0, 1.5, 0.5, -1, 1.5, 1, 0.5, 0\}$ .