

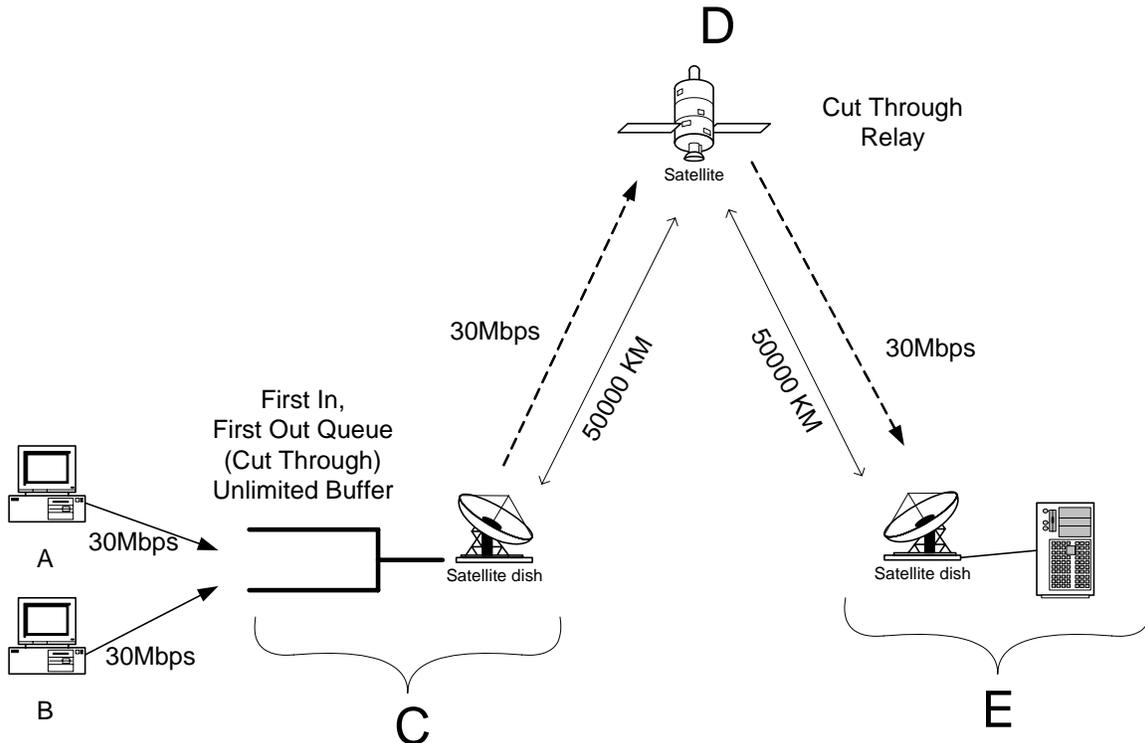
EECS 122: Introduction to Communications Networks

Spring 2004

Assignment 5: Solution Correction for Problem 7

May 18, 2004

Problem 7. [30 points]



Suppose Station **A** has an unlimited amount of data to transfer to Station **E**. Station **A** uses a sliding window transport protocol with a fixed window size. Thus, Station **A** begins a new packet transmission whenever, the number of unacknowledged packets is less than W and any previous packet being sent from **A** has finished transmitting.

The size of the packets is 10000 bits (neglect headers). So for example if $W > 2$, station **A** would start sending packet 1 at time $t = 0$, and then would send packet 2 as soon as packet 1 finished transmission, at time $t = 0.33\text{ms}$. Assume that the speed of light is $3 \times 10^8 \text{ m/s}$.

- (a) Suppose station **B** is silent, and that there is no congestion along the acknowledgement path from **C** to **A**. (The only delay acknowledgements face is the propagation delay to and from the satellite.) Plot the average throughput as a function of window size W . What is the minimum window size that **A** should choose to achieve a throughput of 30Mbps? Call this value W^* . With this choice of window size, what is the average packet delay (time from leaving **A** to arriving at **E**)? [10 points]

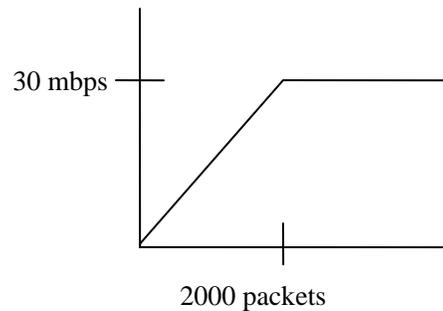
$$[\text{Round Trip Prop}] = (2 \times 10^8 \text{ m}) / (3 \times 10^8 \text{ m/s}) = 0.666 \text{ sec}$$

$$\begin{aligned}
W_{\min} &= [\text{Round Trip Prop}] \times [\text{Link Rate}] \\
&= 0.666 \times (3 \times 10^7 \text{ bit/s}) \\
&= 2 \times 10^7 \text{ bits} \\
&= 2 \times 10^7 \text{ bits} / (10^4 \text{ bits / packet}) \\
&= \mathbf{2000 \text{ packets}}
\end{aligned}$$

For values of $W \leq W_{\min}$,

$$[\text{throughput}] = W / [\text{Round Trip Prop}]$$

so throughput grows linearly with respect to W , for $W \leq W_{\min}$. The throughput saturates at the link rate, 30 mbps for $W \geq W_{\min}$. Thus the throughput plot looks like:



- (b) Suppose now that station **B** also has an unlimited amount of data to send to **E**, and that station **B** and station **A** both use the window size W^* . What throughput would **A** and **B** get for their flows? How much average delay do packets of both flows incur? [5 points]

The flows have more than adequate window sizes to keep the pipe full, so the total throughput will be 30 mbps. Because each flow has the same window size, by symmetry, each flow will get **15mbps**.

We will use Little's result to get delay. Recall that Little's result says that:

$$[\text{Avg occupancy}] = [\text{Avg Arrival rate}] \times [\text{Avg Delay}]$$

Applying the result in this context we have that:

$$\begin{aligned}
&[\text{Avg \# of outstanding packets of A's in the network}] \\
&= [\text{Avg Arrival Rate of A's pkts}] \\
&\quad \times [\text{Avg time one of A's packet is "outstanding"}]
\end{aligned}$$

Because **A**'s uplink to the network is fast enough to keep its window full, the "avg # of outstanding packets of **A**'s in the network" is just its window size W . Thus we have:

$$\begin{aligned}
W &= [\text{A's Throughput}] \times [\text{A's RTT}] \\
2 \times 10^7 \text{ (bits)} &= 1.5 \times 10^7 \times \text{RTT} \\
\text{RTT} &= 1.333 \text{ seconds.}
\end{aligned}$$

RTT is the sum of the delays on the forward and reverse paths. The delay on the reverse path is 0.333. So the one-way delay of a packet on the forward path is **1.00 seconds**. This will be the delay experienced by A and B's packets.

- (c) What average throughput and delays would **A** and **B** get for their flows if **A** and **B** both used window size $0.5W^*$? What would be the average throughput and delay for each flow if **A** used a window size of W^* and **B** used a window size of $0.5W^*$. [5 points]

i) When each has window size $0.5W^*$, the two flows have combined window sizes that are sufficient to keep the pipe full. Their combined throughput will be 30 mbps, so by symmetry each will get **15 mbps**. The delay can be found by:

$$RTT = W / [\text{Throughput}] = 10^7 / (1.5 \times 10^7) = 0.666.$$

Subtracting the delay on the reverse path, we get a forward path delay of **0.333 seconds**.

ii) When A's window is W^* and B's is $0.5W^*$ the two flows have combined window sizes that are sufficient to keep the pipe full. Their combined throughput will be 30 mbps. A will have twice as many packets in the pipeline as B, so their throughputs will have a ratio of 2:1. Thus **A gets 20 mbps and B gets 10 mbps**. A's delay is found by:

$$RTT = W / [\text{Throughput}] = (2 \times 10^7) / (2.0 \times 10^7) = 1.00$$

Subtracting the delay on the reverse path, we get a forward path delay of **0.666 seconds for A**. Similarly for B, we get that

$$RTT = W / [\text{Throughput}] = (1 \times 10^7) / (1.0 \times 10^7) = 1.00$$

Giving us a forward path delay of **0.666 seconds for B**.

- (d) Imagine you are station **A** and you would like your flow to have a high throughput and low delay. You quantify your preferences with the following utility function: If R_a is the average throughput of your flow in Mbps, and D_a is your average delay in seconds, you want to maximize $J_a(R_a, D_a)$ where

$$J_a(R_a, D_a) = R_a - 3D_a.$$

You have no regard for how well **B**'s flow performs. Station **B**, will either pick a window of W^* or $0.5W^*$, but you won't know which he picks until after you decide your window size. Should you pick W^* or $0.5W^*$ as your window size? [5 points]

Use the notations (A's Window, B's Window) \rightarrow A's throughput, A's Delay, J_a

$(W^*, 0.5W^*) \rightarrow$ 20 mbps, 0.666 seconds, $J_a=18$

$(0.5W^*, 0.5W^*) \rightarrow$ 15 mbps, 0.333 seconds, $J_a=14$

$(W^*, W^*) \rightarrow$ 15 mbps, 1.000 seconds, $J_a = 12$

$(0.5W^*, W^*) \rightarrow$ 10 mbps, 0.666 seconds, $J_a = 8$

When B uses window $0.5W^*$, A's best response is to use a window of W^* .

When B uses window W^* , A's best response is also to use a window of W^* .

A should use a window of W^* even if A does not know how B will play.

- (e) Assume that station **B** picked the same window as you did in part (d). What would be the combined value of both of your utilities? The combined utility has the expression:

$$J_a(R_a, D_a) + J_b(R_b, D_b) = R_a + R_b - 3D_a - 3D_b.$$

Now suppose a central planner picked your and your opponent's window sizes. Could the planner achieve a higher combined utility? If so, what value could the planner achieve? [5 points]

(Hint: compare different combinations of window sizes for **A** and **B**:

$(0.5W^*, 0.5W^*)$, $(0.5W^*, W^*)$, etc.)

If each player uses a window of W^* , each player has a utility of 12, so the **sum is 24**.

The combination $(0.5W^*, 0.5W^*)$ achieves a combined utility of 28, which is the highest value a central planner can achieve.