Optimal FIR Filter Design

For low pass filter, alternations always occur at $w_p$ and $w_s$.

Fig 7.38

Filter will be equi-ripple except possibly at $w=0$ or $w=\pi$.

Fig 7.39.

slope $0$
Alg. for Optimal Filter Design

Parks/McClellan:

Given $w_s$, $w_p$, $K = \frac{\delta_1}{\delta_2}$, $M$

find $a_i(n)$ to minimize $\delta_2$

Problem B

$G(\omega) = \sum_{n=0}^{M} a_n \cos(\omega n)$

$E(w_i) = \pm \delta_2$

$E(w_i) = -E(w_{i+1})$

$W(w_i) \left[ G(w_i) - D(w_i) \right] = (-1)^{i+1} \delta_2$

$wi = alternating frequencies$
Show "hard": show, if I know \( w_i \), how to find \( a(n) \) and \( s_2 \).

\[
G(w_i) = \sum_{n=0}^{M} a(n) \cos(w_i \cdot n)
\]

\[
G(w_i) = \frac{(-1)^{i+1} s_2}{W(w_i)} + D(w_i)
\]

\[
\sum_{n=0}^{M} a(n) \cos(w_i \cdot n) = \frac{(-1)^{i+1} s_2}{W(w_i)} + D(w_i)
\]
\[ i = 1 \]
\[ a(0) \cos(w_0 0) + a(1) \cos(w_0 1) + a(2) \cos(w_0 2) + \cdots \]
\[ = (-1)^{i+1} \delta_2 + D(w_i) \]
\[ i = 2 \]
\[ a(0) \cos(w_2 0) + a(1) \cos(w_2 1) + \cdots \]
\[ = \left( \begin{array}{c}
\end{array} \right) \]
\[ i = M+2 \]
\[ \Rightarrow M+2 \text{ linear equations in } a(n) \]

\[ M+1 \text{ unknown } a(n) + \delta_2. \]

Solve a linear syst of Eqs. in \( M+2 \) unknowns.
\[
\begin{align*}
&+ \\
= & \frac{1}{s} \\
\begin{bmatrix}
(a(0)) & (a(1)) & \cdots & (a(m)) \\
(a(0)) & (a(1)) & \cdots & (a(m)) \\
\end{bmatrix} \\
\end{align*}
\]

Given if we know

\( Y \)
Remes exchange algorithm.

$P \& M$: showed, $\delta_2$ is given by the following expression if $w_i$ are known:

$$
\delta_2 = \frac{\sum_{k=1}^{M+2} b_k D(w_k)}{\sum_{k=1}^{M+2} b_k (-1)^{k+1}}
$$

when $b_k = \prod_{i=1, i \neq k}^{M+2} \frac{1}{\cos(w_i)}$.
Empirical studies

Approx. length of filter = 1 - \( \frac{10 \log_{10} (\delta_1 \delta_2) - 13}{2.3 \Delta W} \)

\[ \Delta W = W_s - W_p \]

Kaiser window length \( \leq 1 + \frac{A - 8}{2.2 \Delta W} \)

A = \(-20 \log_{10} \delta \)

\[ W_s = 0.677 \]

\[ \delta_1 = 0.01 \]

\[ \delta_2 = 0.001 \]

Optimum filter \( N = 2T = 2M + 1 \)

Kaiser filter \( N \approx 38 \)