Using DFT for filtering only long sequences

\[ X(n) \xrightarrow{h(n)} y(n) \]

Exploit linearity property of convolution:

\[ [X_1(n) + X_2(n)] \ast h = (X_1 \ast h) + (X_2 \ast h) \]
Overlap Add

Segment only then goes +1

L

L

L

L

L

L

L

L

L
2) Convolve each chunk with \( h \) for \((L + P - 1)\) new point.

3) Add up all the convolutions.

\[ \text{Overlap Save} \quad L \gg P \]

Thought experiment:

- Correct: Pad with zeros to get \( L + P - 1 \) point sequence.
  - Multiply \( 2 \times L + P - 1 \) point DFTs.
  - Take \( L + P - 1 \) point IDFT.

- Suppose I take \( L \) point DFT of \( X_1 \) & \( X_2 \).
  - Multiply two \( L \) point DFTs.
Take inverse L point IDFT.

What goes wrong? Can show only the first \( p-1 \) points are wrong. Rest are correct.

Why? Sketchy illustration.
padding with zero

not padding with zeros

X1(n)

X2(n) P point

X3(1-n)
(3) Theorem: Given a right triangle by the point of intersection of each such line.

Let point of intersection be \( A \) and \( B \).

Motivation: Let point of intersection be \( C \) and \( D \).

(2) Let point of intersection be \( E \) and \( F \).

Sketch: Sketch with each other by P-I point.

(1) Sketch: Sketch with each other by L point to match each other.
Fig 8.23 of 025
0.24 to 025.
DCT = Discrete Cosine Transform
+ Compression property
  >
  Pixel domain coding

+ DCT used standard

JPEG,
MPEG 1
2
4,
H.261, H.263, H.263+
H.263++, H.264, ...
Other than IPEB 2000, uses wavelets. Every other image/video stand uses ACD.

- Compute vision
- Multi-resolution

- Haar wavelet
- Analyze

- Filter bank
- Signal processing

- Mallet
- Multiresolution

Wavelets
DFT: \( x(n) \) \( \xrightarrow{\text{DFT}} \) \( X(k) \) over periodic DFT

\[ \frac{1}{N} \sum_{k=0}^{N-1} x(n) e^{-j2\pi nk/N} \]

Why DCT?

\[ X(k) \]

DCT: \( x(n) \) \( \xrightarrow{\text{DCT}} \) \( X(k) \) over periodic DCT

\[ \frac{1}{N} \sum_{k=0}^{N/2} x(n) \cos \left( \frac{2\pi nk}{N} \right) \]

x(n)

Real

DFT

X(k)

Real

Complex

\( n \), Hartley, Hadamard, Walsh ...
4 Kinds of DCT

We'll focus on DCT-2.

\[ X(k) = 2 \sum_{n=0}^{N-1} x(n) \cos \left( \frac{2\pi (2n+1)k}{2N} \right) \]

Synthetic:

\[ x(n) = \frac{1}{N} \sum_{k=0}^{N-1} \beta(k) X(k) \cos \left( \frac{2\pi (2n+1)k}{2N} \right) \]

\[ \beta(k) = \begin{cases} \frac{1}{2} & k = 0 \\ 1 & 1 \leq k < N \end{cases} \]
The relation between DFT of $x(n)$ and its DCT-2 is:

**Proposal**: 1. Start with $N$ pt real seq $x(n)$
2. Pad it with $N$ zeros $\rightarrow x_{2N}(n)$
3. Form a periodic seq $\hat{x}_c(n) = x_{2N}(n) + x_{2N}(-n-1)$
4. Take $2N$ pt DFT of one period of $\hat{x}_c(n) \rightarrow \hat{x}_2(k)$