DISCUSSION #1 Tuesday Sept 6th

Problem 2.39

a) The system is <u>not</u> time invariant

Let $x_1[n] = \delta[n]$ $\qquad$ $y_1[0] = 1$, $y_1[1] = x_1[1] + a\,y_1[0] = a$

$x_2[n] = \delta[n-1]$ $\qquad$ $y_2[0] = 1$, $y_2[1] = x_2[1] + a\,y_2[0] = 1+a$.

$y_2[n]$ is not $y_1[n]$ shifted by 1.

b) the system is <u>not</u> linear

$x[n] \rightarrow \boxed{} \rightarrow y[n]$, $y[0] = 1$

$2x[n] \rightarrow \boxed{} \rightarrow y'[n]$, $y'[0] = 1 \neq 2y[0] = 2$

c) let $n > 0$.
$$\begin{aligned} y[n] &= x[n] + a\,y[n-1] \\ &= x[n] + a\,x[n-1] + a^2 y[n-2] \\ &= x[n] + a\,x[n-1] + a^2 x[n-2] + \cdots + a^{n-1} x[1] + \overbrace{a^n y[0]}^{0} \\ &= \sum_{k=1}^{n} a^{n-k} x[k] \end{aligned}$$

let $x_3[n] = \alpha x_1[n] + \beta x_2[n]$.
$$y_3[n] = \sum_{k=1}^{n} a^{n-k} x_3[n] = \sum_{k=1}^{n} a^{n-k}\left(\alpha x_1[n] + \beta x_2[n]\right) = \alpha \sum_{k=1}^{n} a^{n-k} x_1[n] + \beta \sum_{k=1}^{n} a^{n-k} x_2[n]$$
$$= \alpha\, y_1[n] + \beta\, y_2[n]$$

<u>$n=0$</u> $\quad y_3[0] = 0 = \alpha\, y_1[0] + \beta\, y_2[0]$

<u>$n<0$</u>
$$\begin{aligned} y[n] = \frac{1}{a}\left(y[n+1] - x[n+1]\right) &= -\frac{1}{a} x[n+1] + \frac{1}{a} y[n+1] \\ &= -\frac{1}{a} x[n+1] - \frac{1}{a^2} x[n+2] + \frac{1}{a^2} y[n+2] \\ &= -\sum_{k=n+1}^{-1} \frac{1}{a^{n-k}} x[k] \end{aligned}$$

Similarly $y_3[n] = \alpha\, y_1[n] + \beta\, y_2[n]$ for $n < 0$.

Thus $\forall n$, $y_3[n] = \alpha y_1[n] + \beta y_2[n]$ $\quad$ The system is linear

d) The system is not time invariant.

Try the same counter-example as in question a.