Follow-up on discussion session 11/14/05

Optimal Type I lowpass filter

Question: when is the number of alternations $L+2$ and when is it $L+3$?

Look at Figure 7.37(a): we have alternations at $0, \omega_p, \omega_s, \pi$ $\Rightarrow L+3$

7.37(d): ______________________ $\Rightarrow L+2$

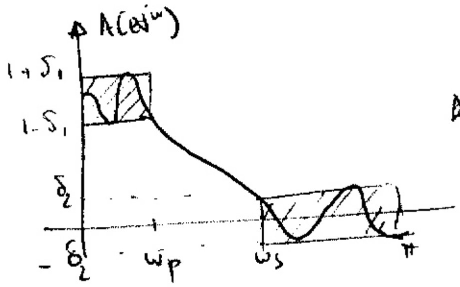
The question we are interested in is why those two cases are different, although they look exactly the same ...

Alternation theorem: $P(x) = \sum_{k=0}^{L} a_k x^k$ is optimal if $E_P(x)$ exhibits at least $(L+2)$ alternations (p. 489)

When doing filter design, we have polynomials where the variable is ω:

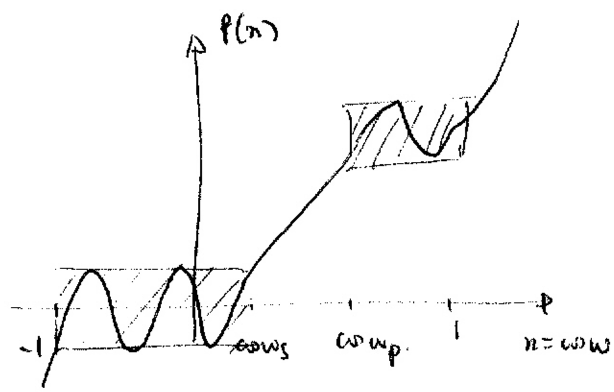
$P(\cos\omega) = \sum_{k=0}^{L} a_k (\cos\omega)^k$. We can apply the alternation theorem to $P(\underline{x})$ where $x = \cos\omega$, since the alternation theorem works for polynomials in x.

Thus, if our specs are:



see in the book p. 493

To $A(e^{j\omega})$ corresponds $P(x)$ such that:



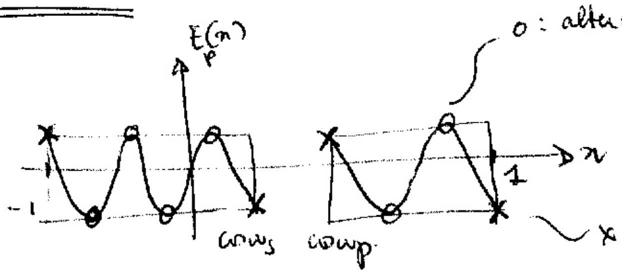
A crucial point is that $P(\cos\omega)$ has 0 slope at $\omega = 0$ and π, but this is not necessarily the case for $P(x)$.

Indeed $\dfrac{dP(\cos\omega)}{d\omega} = -\sin\omega \, P'(\cos\omega) \Rightarrow \left.\dfrac{dP(\cos\omega)}{d\omega}\right|_{\omega=0 \text{ or } \pi} = 0$

Let's now look at various possible cases for the alternations of $P(x)$.

$P(x)$ is of degree L, so it has $L-1$ points of 0 slope, and they all correspond to alternations. There are alternations at $\cos\omega_s$ and $\cos\omega_p$, and they do not have 0 slope. However, the alternations at -1 and 1, if there are any, may or may not have 0 slope.

Case $L+3$:



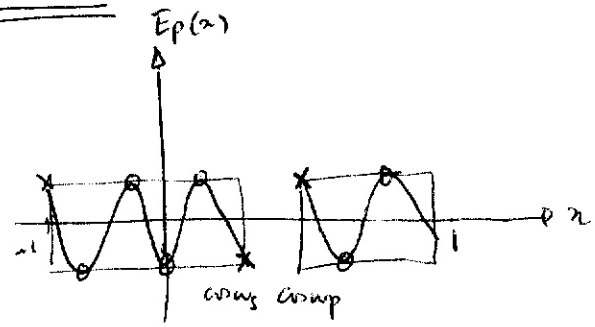
o : alternations with slope 0 → $L-1$ of them

x : alternations with slope $\neq 0$ → 4 of them

→ $L+3$

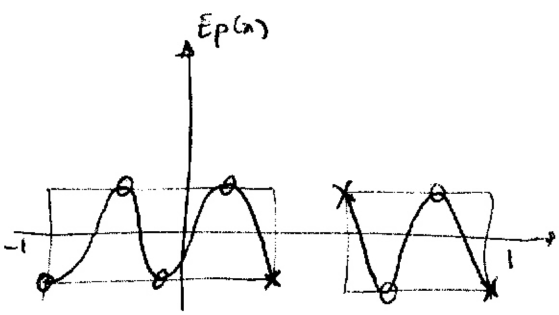
This corresponds to figure 7.37a. See that the alternations at -1 and 1 correspond to alternations at $\omega=0$ and $\omega=\pi$ with 0 slope for $P(\cos\omega)$, but with nonzero slope for $P(x)$.

Case $L+2$



$L-1$ o's, 3 x's] ⇒ $L+2$ alternations

→ 7.37b



Here, a point of zero slope is at $x=-1$!

We have $L-1$ o's, 3 x's] ⇒ $L+2$ alternations, thus $P(x)$ is optimal.

This case correspond to 7.37d. It was hard to tell just by looking at 7.37d that $P(x)$ had a zero at $x=-1$, but it is what is going on.

By knowing L and looking at figure 7.37, we could tell that those cases were optimal since we were able to count $L+2$ or $L+3$ alternations.

I hope this clarifies the point! Please come to office hours if it is still unclear!

Thanks! Pierre