EE123 Review Session Wednesday Oct 19th

Here is what I did for those of you who couldn't make it !

- Problem 6 in Midterm I Review

Clarification for the solution of (a)

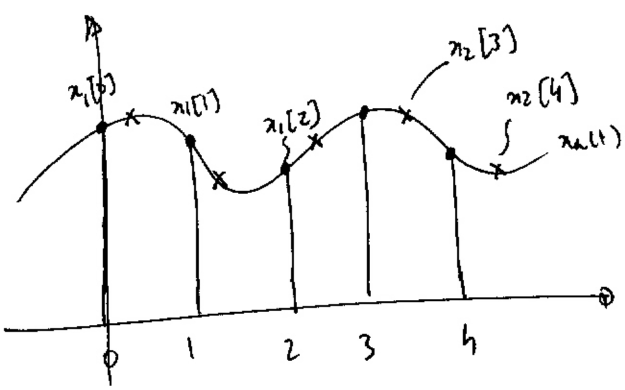
sample $x_a(t)$ at $T=1 \to x_1[n] = x_a(n)$

— $\underbrace{x_a(t+\frac{1}{4})}$ — $\to x_2[n] = x_a(n+\frac{1}{4})$

call this $y_a(t)$. It is a shift in time of $x_a(t)$ (no modulation in frequency domain)

$$X_2(\omega) = \sum_{k\in\mathbb{Z}} Y_a(j(\omega + 2\pi k))$$

We have $Y_a(j\Omega) = e^{j\frac{\Omega}{4}} X_a(j\Omega)$, so $X_2(\omega) = \sum_{k\in\mathbb{Z}} e^{j\frac{\omega+2\pi k}{4}} X_a(j(\omega+2\pi k))$



- Problem 4.21

(a) is easy

(b) Look at the formula $X(e^{j\omega}) = \frac{1}{T}\sum_{k\in\mathbb{Z}} X_c\left(j\,\frac{\omega - 2\pi k}{T}\right)$

To obtain $X(e^{j\omega})$, we "stretch" $X_c(j\Omega)$ and "shift it" periodically.

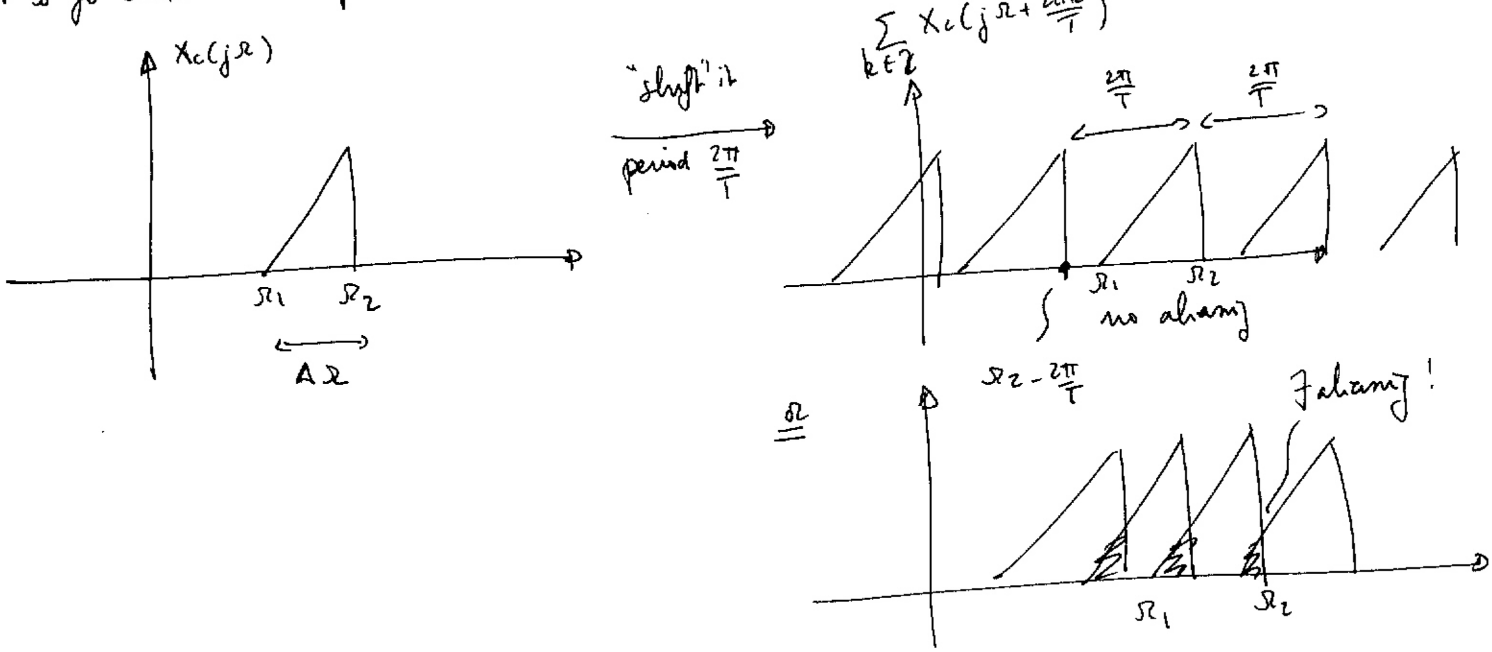
Aliasing occurs in the "shifting" part (if it occurs ...)

$$X_c(j\Omega) \xrightarrow{\text{shift}} \sum_{k\in\mathbb{Z}} X_c\left(j\Omega + \frac{2\pi k}{T}\right) \xrightarrow[\Omega = \frac{\omega}{T}]{\text{stretch}} \sum_{k\in\mathbb{Z}} X_c\left(j\left(\frac{\omega + 2\pi k}{T}\right)\right)$$

potential aliasing happens here

this function is $\frac{2\pi}{T}$ periodic

Let's go back to our pb.



Thus, to avoid aliasing, we need $\Omega_2 - \frac{2\pi}{T} < \Omega_1 \Rightarrow \boxed{\frac{2\pi}{T} > \Delta\Omega}$

- <u>Discrete-time processing of continuous time system</u>

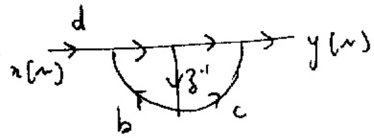
  I basically went over p.155 in the book.

- <u>Quantization</u>

  Given a transfer function $H(z)$, there are many realizations (direct form I, II, cascade, etc...)

  Quantization will affect $H(z)$ differently with respect to various realizations.

<u>Ex</u> $H(z) = \dfrac{z^{-1} - a}{1 - a z^{-1}}$ realization:

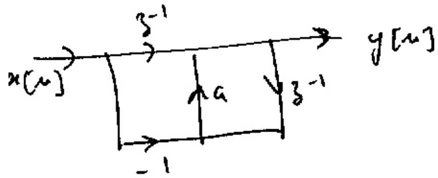


this corresponds to $H(z) = \dfrac{dc\, z^{-1} + d}{1 - b z^{-1}}$

So take $d = -a$, $b = a$, $c = \frac{-1}{a}$ to get $\dfrac{z^{-1} - a}{1 - a z^{-1}}$

After quantization, get $H_Q(z) = \dfrac{\hat{d}\hat{c}z^{-1} + \hat{d}}{1 - \hat{b}z^{-1}}$ 3 different numbers are quantized.

Realization 2 :



After quantization $H_{Q'}(z) = \dfrac{z^{-1} - \hat{a}}{1 - \hat{a}z^{-1}}$

In general, $H_Q(z) \neq H_{Q'}(z)$!

$H_{Q'}(z)$ looks more like $H(z)$ (it is actually an all-pass filter as well)
Thus you might prefer realization 2 to realization 1. However, (2) has 2 delays and (1) only 1 $\longrightarrow$ trade off.