Nov 16, 05

IIR Filter Design. Transformation

Impulse Invariance.

Bilinear. Transf.

Low pass filter.

high pass digital filters.

1 Use LP analog filter

2 high pass analog filter

Approach 1 To design Digital high pass IIR filters

$$\boxed{\begin{matrix}\text{C/T LPF}\\ H_a(s_1)\end{matrix}} \xrightarrow{\text{C/C}} \boxed{\begin{matrix}\text{C/T HPF}\\ H_{a'}(s_2)\end{matrix}} \xrightarrow[\text{Bilinear Transf.}]{\text{C/D}} \boxed{\begin{matrix}H_d(z)\\ \text{D.T.}\\ \text{HPF.}\end{matrix}}$$

C = Continuous.

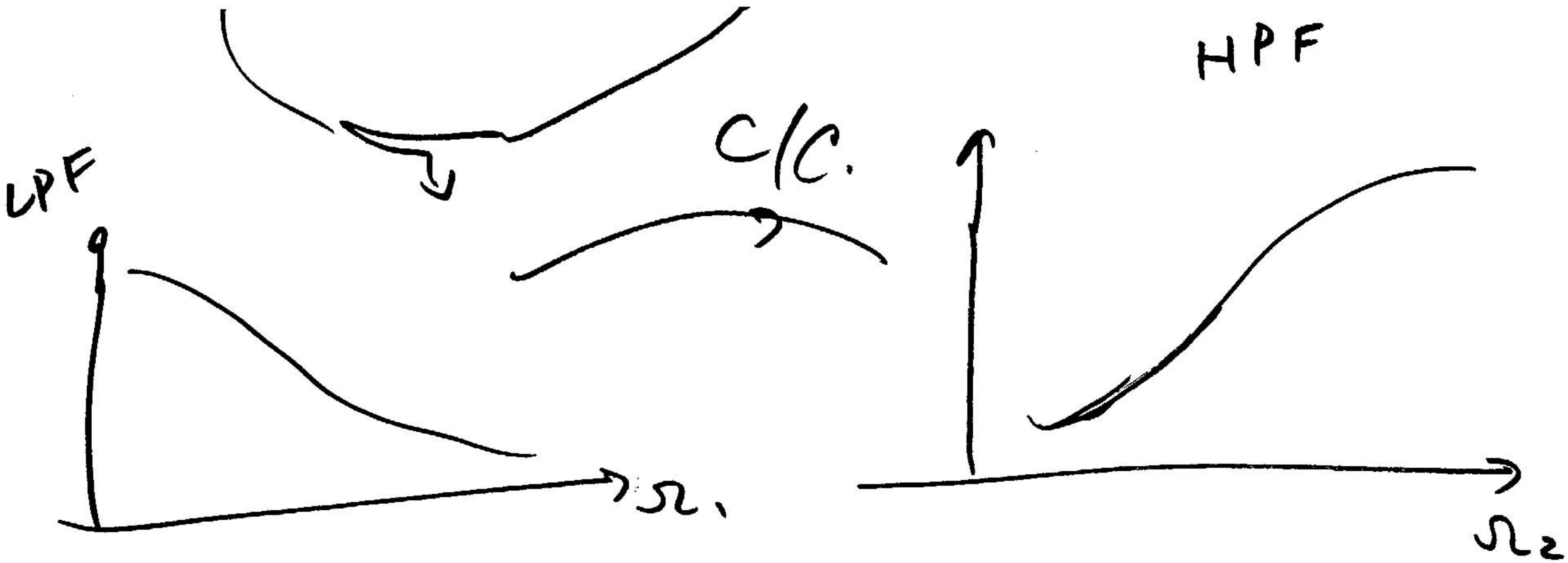
a, a' = analog.

d. = discrete = D

T = Time.

HPF = high pass filter

LPF = low pass filter.



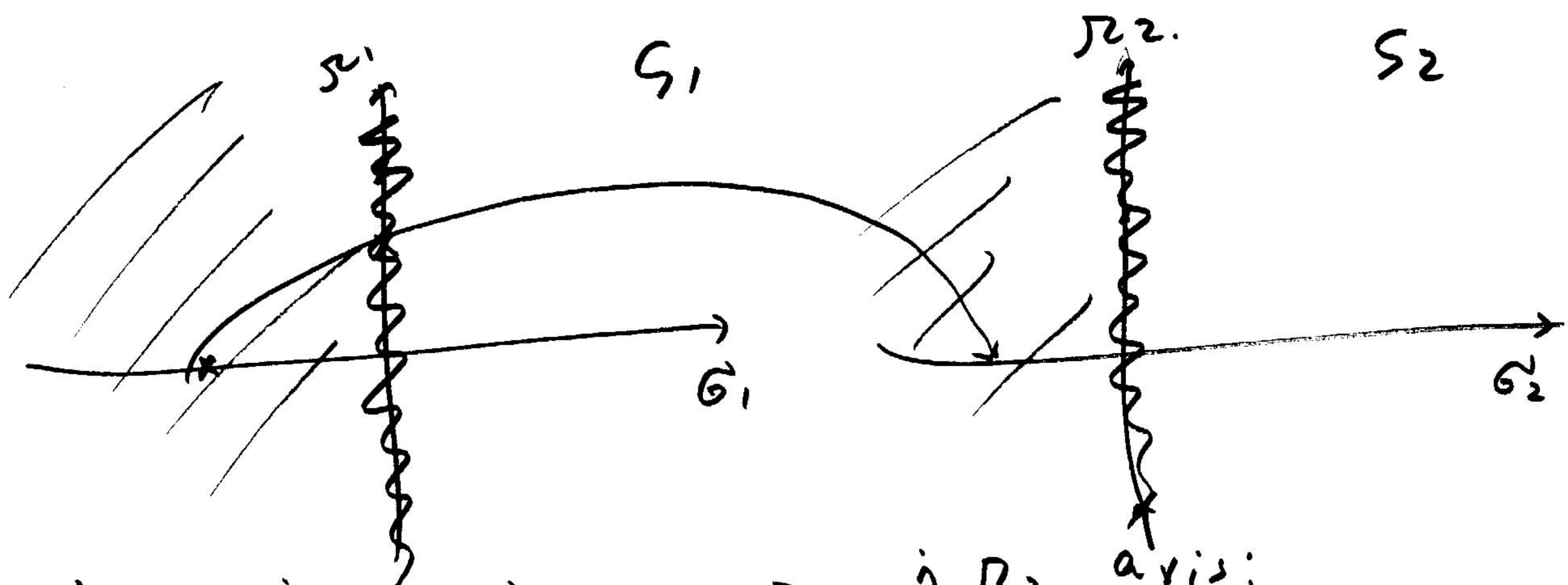
$$H_a(s_1) \xrightarrow{C/C\,??} H_{a'}(s_2)$$

LPF HPF.

C/C: Propose

$$H_{a'}(s_2) = \left[H_a(s_1) \right]_{s_1 = \frac{\Omega_u}{s_2}}$$

Ω_u = positive constant.



Show $j\Omega_1$ axis $\longrightarrow$ $j\Omega_2$ axis:

Let $s_1 = j\Omega_1 = \dfrac{\Omega_u}{s_2} \Rightarrow s_2 = \dfrac{\Omega_u}{j\Omega_1} = \underbrace{-j\dfrac{\Omega_u}{\Omega_1} = \sigma_2 + j\Omega_2}$

$\Rightarrow \boxed{\sigma_2 = 0, \quad \Omega_2 = -\dfrac{\Omega_u}{\Omega_1}}$ Yes: $j\Omega_1 \rightarrow j\Omega_2$.

Show: LHPlane in S_1 maps onto LH Plane in S_2. ? yes

causal + stable S_1 $\longrightarrow$ causal + stable in S_2

~~[illegible]~~ LH plane in s_1 $\longrightarrow$ LHP in s_2

Translation Specs:

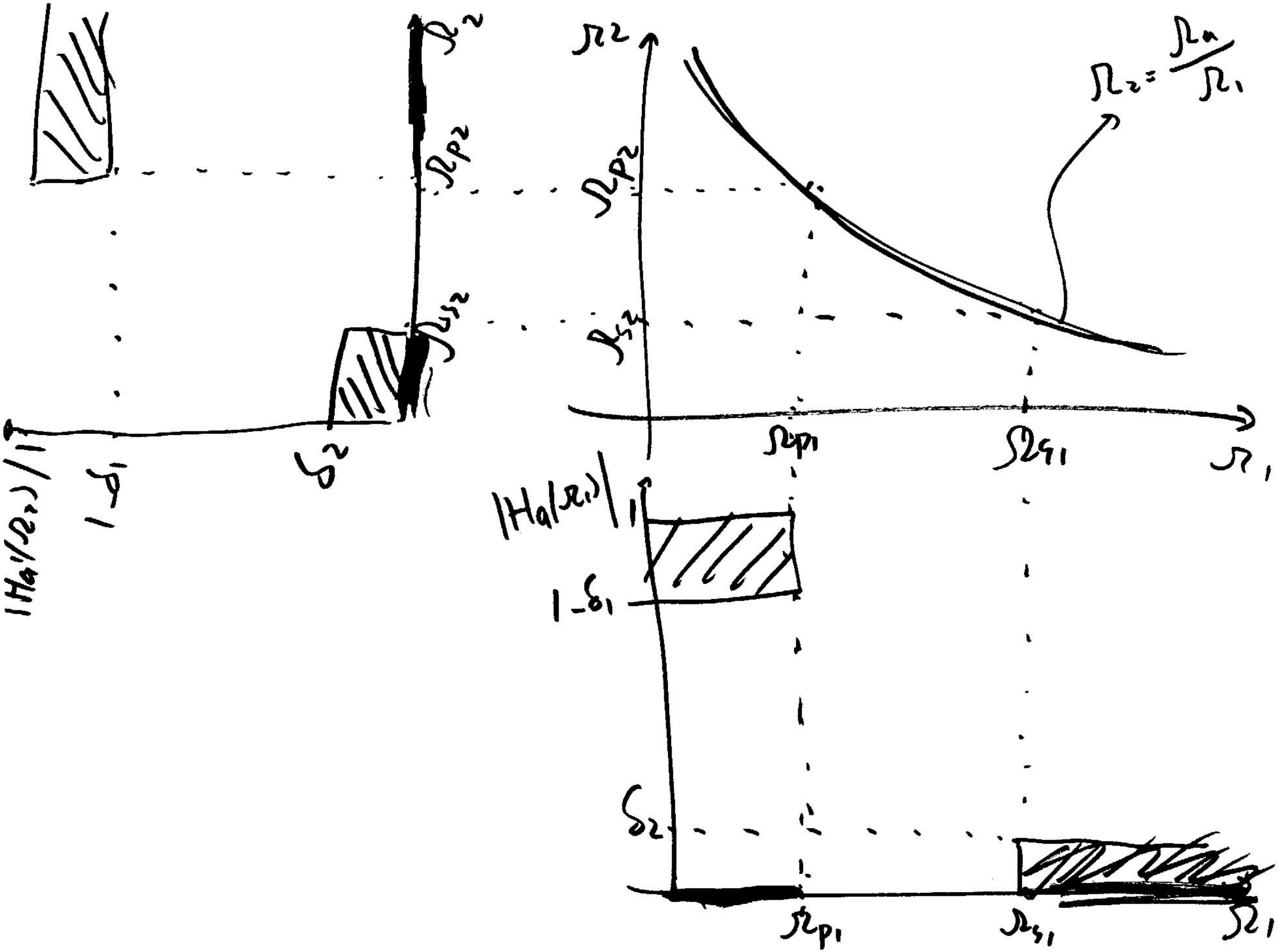
$$\left.\begin{aligned} |H_a'(\Omega_2)| &= \left| H_a(\Omega_1) \right|_{\Omega_1 = -\frac{\Omega_u}{\Omega_2}} \\ |H_a(\Omega_1)| &= |H_a(-\Omega_1)| \end{aligned}\right\} \Longrightarrow$$

assume $h_a(t)$ real $\Longrightarrow$ $|H_a(\Omega_1)| = |H_a(-\Omega_1)|$

$\Rightarrow$ Freq. Res. mag. is symmetric

$$|H_a'(\Omega_2)| = \left| H_a(-\Omega_1) \right|_{\Omega_1 = -\frac{\Omega_u}{\Omega_2}} \Longrightarrow$$

$$|H_a'(\Omega_2)| = \left| H_a(\Omega_1) \right|_{\Omega_1 = \frac{\Omega_u}{\Omega_2}}$$



$\Omega_2 = \frac{\Omega_a}{\Omega_1}$
Ω_2
Ω_{p2}
Ω_{s2}
Ω_{p1}
Ω_{s1}
Ω_1
$|H_a(\Omega_1)|$
1
$1-\delta_1$
δ_2
$|H_a(\Omega_2)|$
δ_2
$1-\delta_1$

Steps To design IIR Digital Filter using Approach 1.

1. Start spec in D.T. domain for HPF = high pass filter

2. Translate specs D.T. HPF ⟶ C.T. HPF.

3. Translate specs C.T. HPF ⟶ C.T. LPF.

4. Design C.T. LPF { Butterworth, Chebyshev, elliptic.

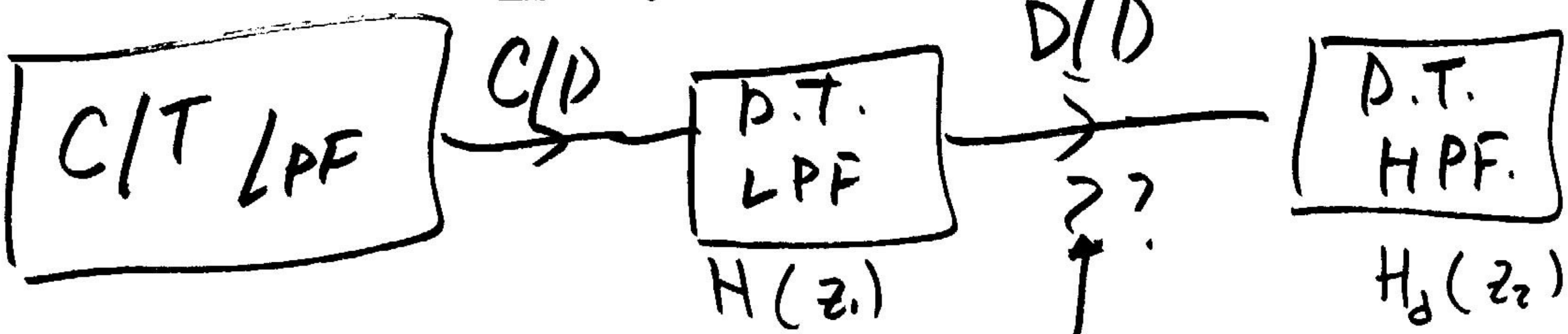
5. Transform C.T. LPF ⟶ C.T. HPF.

6. Transform C.T. HPF ⟶ D.T. HPF.

Approach #2

Design DT. IIR HPF.

Transformation



Proposal:

$$H_d(z_2) = \left[H(z_1) \right]_{z_1 = -\frac{\alpha + z_2}{1 + \alpha z_2}}$$

α = real. $|\alpha| < 1$

Does it map $e^{j\omega_1} \longrightarrow e^{j\omega_2}$ in z_2?

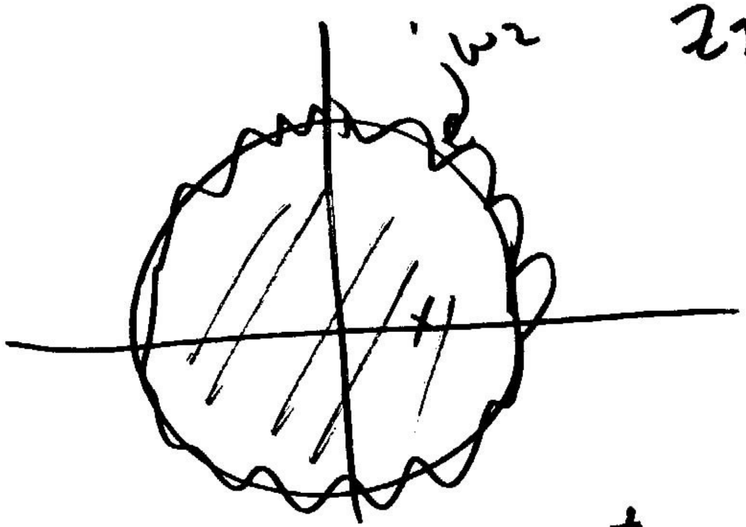
Let $z_2 = e^{j\omega_2}$ in z_1 $\Longrightarrow$

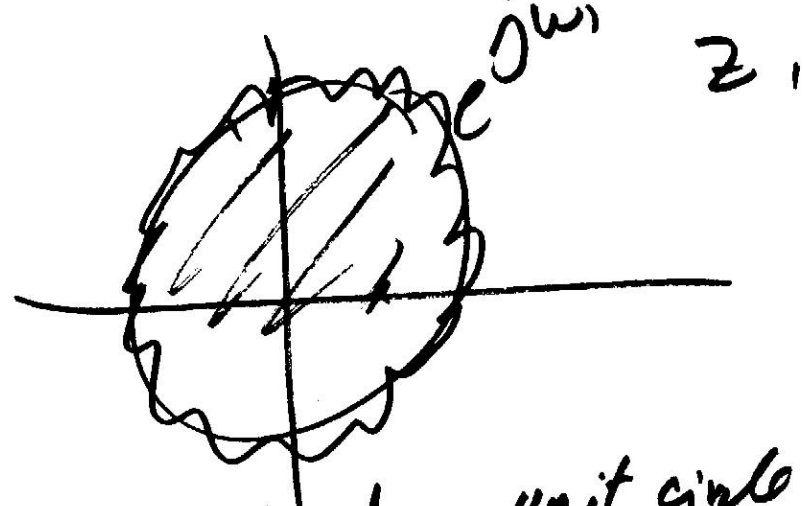
$$z_1 = -\frac{\alpha + e^{j\omega_2}}{1 + \alpha e^{j\omega_2}} = -\frac{e^{j\omega_2}(1 + \alpha e^{-j\omega_2})}{(1 + \alpha e^{j\omega_2})}$$

$$|z_1| = |-1|\,|e^{j\omega_2}|\,\frac{|1 + \alpha e^{-j\omega_2}|}{|1 + \alpha e^{j\omega_2}|} = 1$$

$\Longrightarrow$ unit circle in z_1

Yes $e^{j\omega_2} \longrightarrow e^{j\omega_1}$





inside unit circle in z_2 $\longrightarrow$ inside. unit circle in z_1

$\Rightarrow$ causal + stable HPF $\longrightarrow$ causal stable LPF.

$$e^{j\omega_1} = -\frac{\alpha + e^{j\omega_2}}{1 + \alpha e^{j\omega_2}} \quad \text{mapping.}$$

$$\omega_1 = f(\omega_2, \alpha)$$

show: if $\omega_1 = f(\omega_2, \alpha) \Longrightarrow$

$\omega_1 = f(-\omega_2, \alpha)$

$$\left[H_d(\omega_2) \right] = \left[H(\omega_1) \right]_{\omega_1 = f(\omega_2, \alpha)}$$

Assume H_d has real impulse response $\Longrightarrow$

$$|H_d(\omega_2)| = |H_d(-\omega_2)|$$

$$|H_d(-\omega_2)| = \underbrace{|H_d(\omega_2)| = |H(\omega_1)|_{\omega_1 = f(-\omega_2, \alpha)}}$$

Suppose $\alpha = 0$

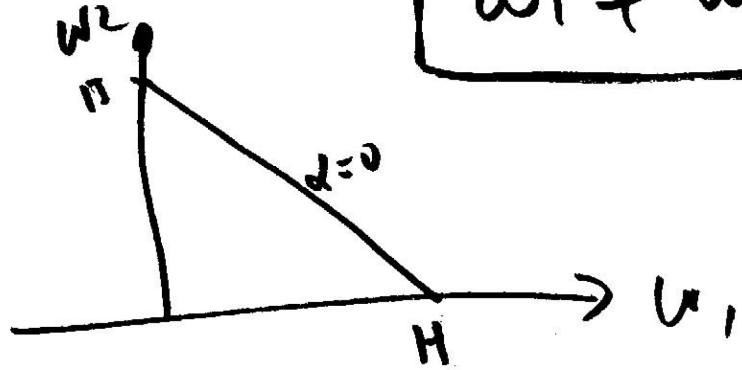
$$e^{jw_1} = -\frac{\alpha + e^{jw_2}}{1 + \alpha e^{jw_2}}$$

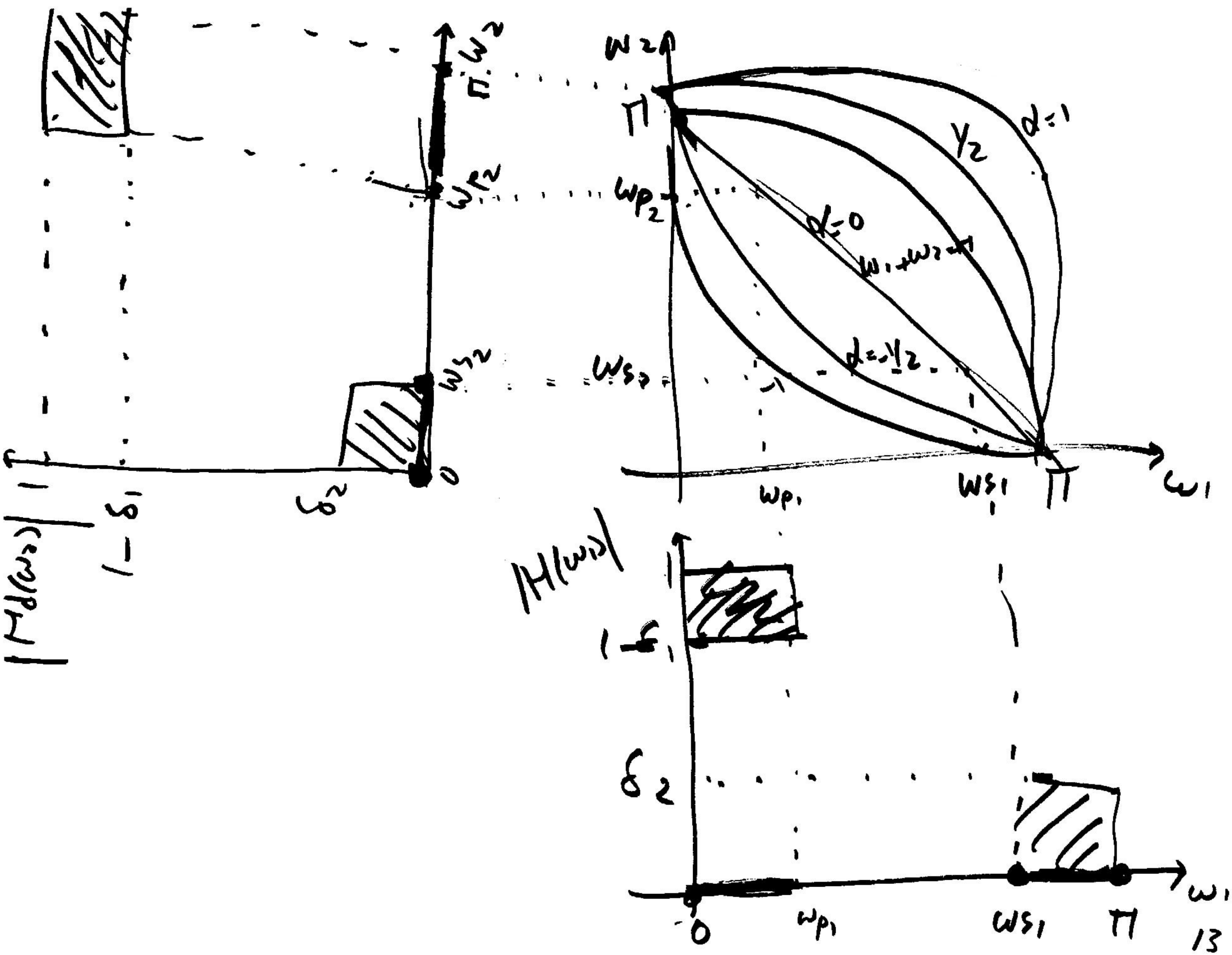
$$e^{jw_1} = -e^{jw_2} = e^{j\pi} e^{jw_2}$$

$$w_2 + \pi = w_1 = f(w_2, \alpha)$$

$$\Rightarrow f(-w_2, \alpha) = -w_2 + \pi = w_1 \Rightarrow$$

$$\boxed{w_1 + w_2 = \pi}$$





ω_2
π
ω_{p_2}
ω_{s_2}
ω_1
ω_{p_1}
ω_{s_1}
$\alpha=1$
$1/2$
$\alpha=0$
$\omega_1+\omega_2=\pi$
$\alpha=-1/2$
$|H_d(\omega_2)|$
$1-\delta_1$
δ_2
0
$|H(\omega_1)|$
1

Steps for HPF, IIR. Discrete Time using Approach 2 of Bilinear Tr

1. Given specs D.T. HPF.
2. Translate specs DT. HPF $\longrightarrow$ DT LPF.

Today's lecture

3. Design D.T. LPF.

last lecture

(a) * specs DT LPF $\longrightarrow$ C.T. LPF.

(b) + Design C.T LPF

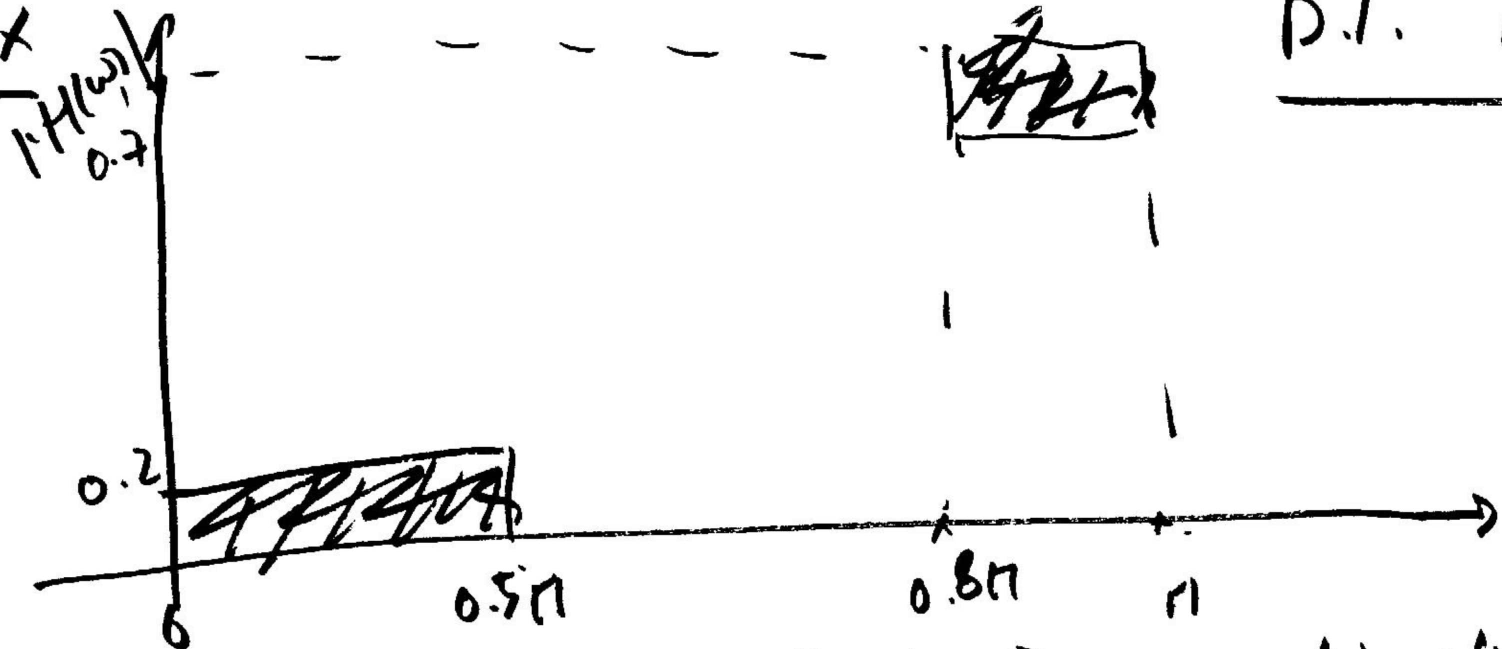
(c) + Transform C.T. LPF $\rightarrow$ DT LPF.

4. Transform DT LPF $\longrightarrow$ D.T. HPF.

$$H_d(z_2) = \left[H(z_1) \right]_{z_1 = -\frac{\alpha + z_2}{1 + \alpha z_2}}$$

D.T. HPF

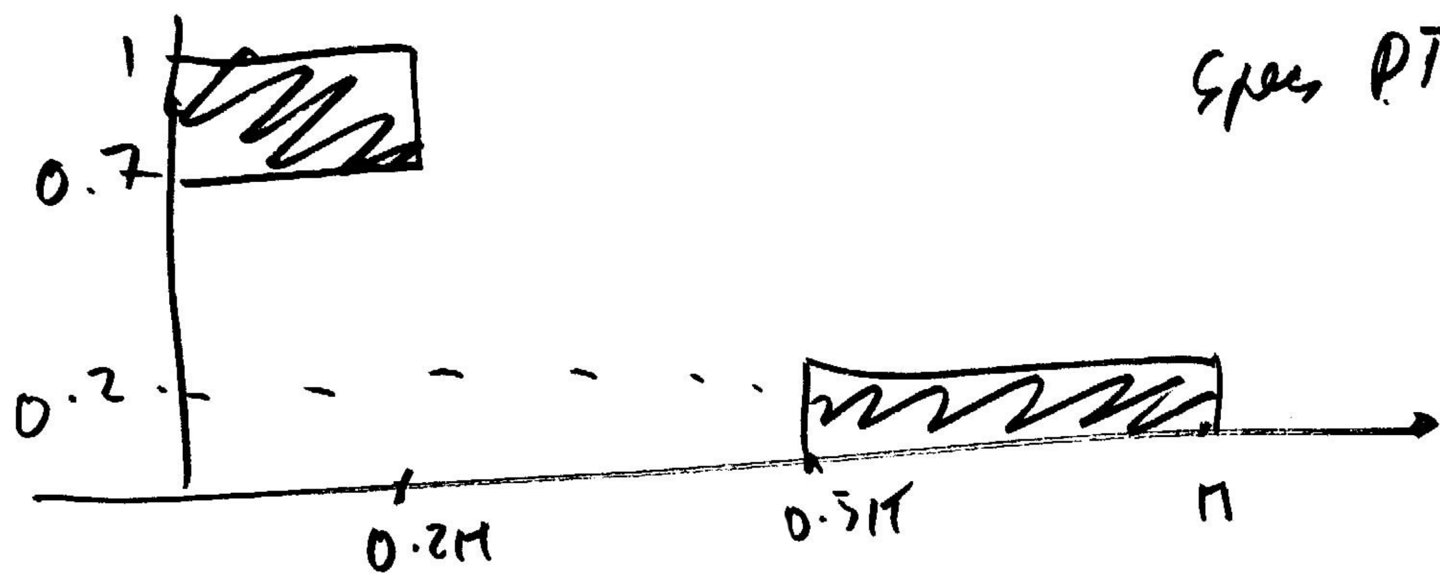
EX



$\omega_1 + \omega_2 = \pi$

- Translate specs onto DT LPF

Specs DT LPF.



- Design LPF, DT.

$$H(z_1) = \frac{0.2416\ z_1^{-1}}{1 - 1.16 z_1^{-1} + 0.414 z_1^{-2}}$$

- Transform LPF D.T $\longrightarrow$ HPF D.T.

$$H_d(z_2) = \frac{-0.2416\ z_1^{-1}}{1 + 1.16 z_2^{-1} + 0.414 z_2^{-2}}$$