

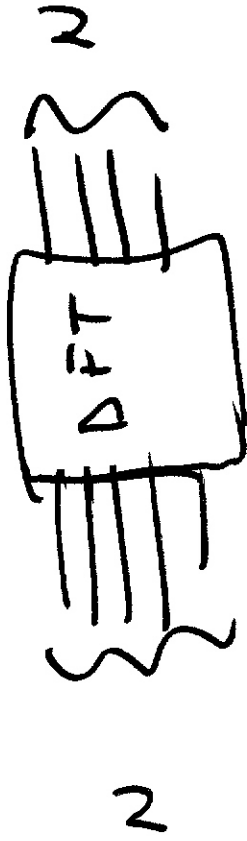
Nov. 16, 05

# DFT = Discrete Fourier Transform

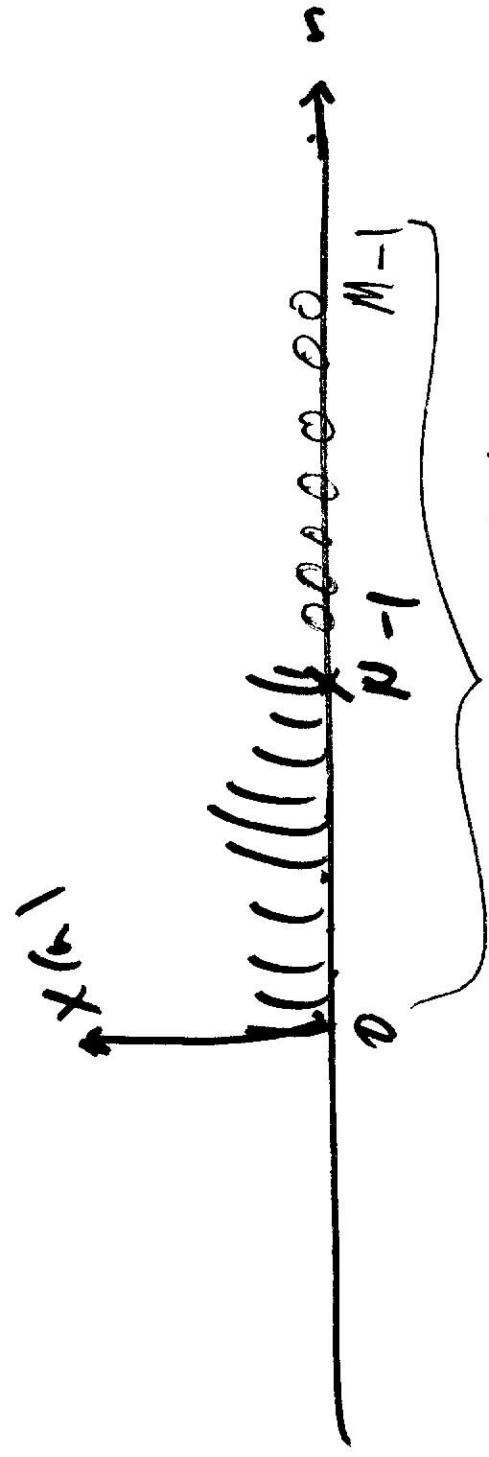
- finite length  $N$  PT seq  $x(n)$   $N$  non zero values.

$$\text{DTFS } \{x(n)\} = X(k) = \begin{cases} \sum_{n=0}^{N-1} x(n) e^{-j2\pi nk} / N & 0 \leq k < N \\ 0 & \text{otherwise.} \end{cases}$$

otherwise.



- Possible To pad an  $N$  PT seq. with  $N$  nonzero values to  $M$  points, ie pad it with  $M-N$  additional zeros
- Possible To Talk about DFT of  $M$  PT seq.

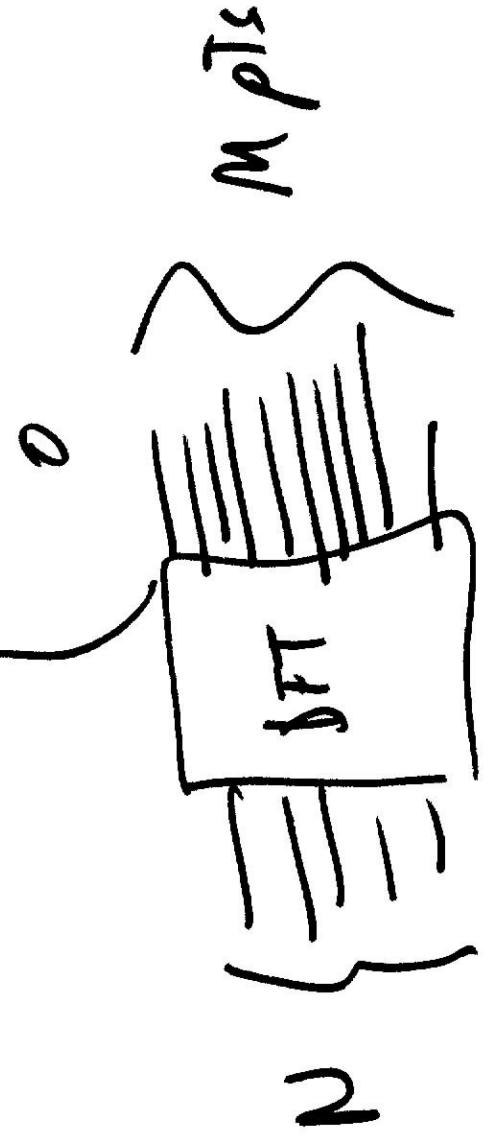


- Can take M PT DFT of  $x(n)$  :

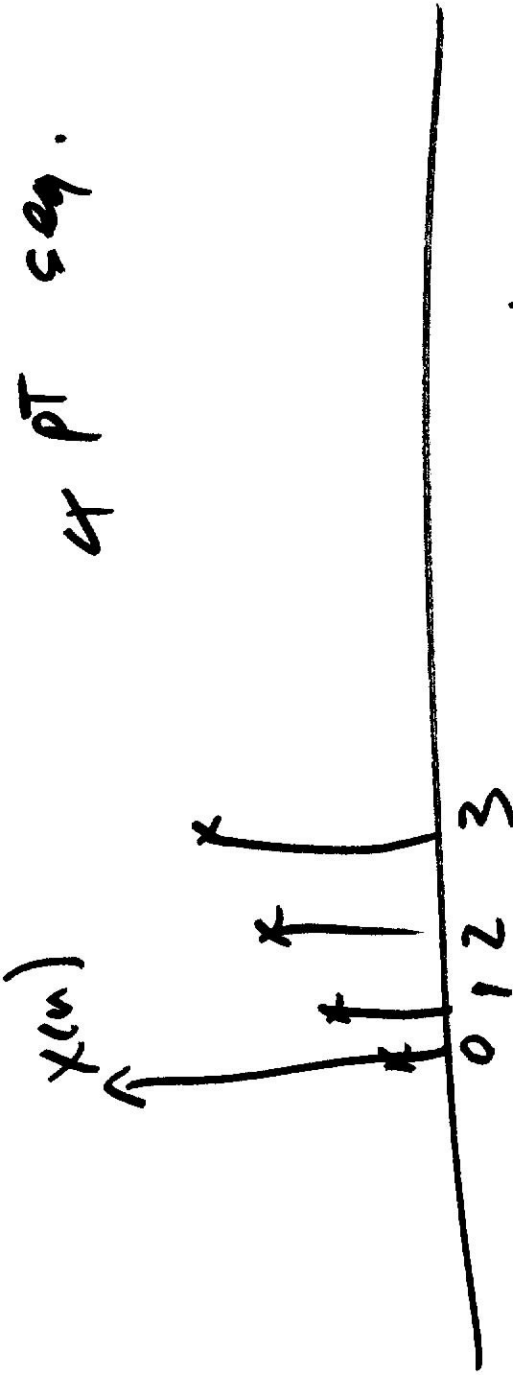
$$X(k) = \sum_{n=0}^{M-1} x(n) e^{-j2\pi nk/M}$$

$$0 \leq k < M$$

otherwise.



Ex



4 #s.

$$4 \text{ pt DFT } x(n) : \sum_{n=0}^3 x(n) e^{-j 2\pi n k / 4}$$

5 #s

$$\sum_{n=0}^4 x(n) e^{-j 2\pi n k / 5}$$

1000 #s

$$5 \text{ pt DFT } x(n) : \sum_{n=0}^4 x(n) e^{-j 2\pi n k / 1000}$$

8 #s

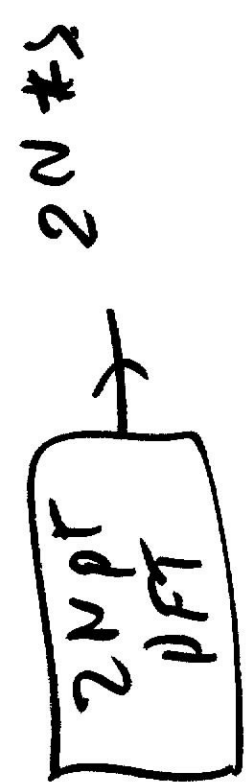
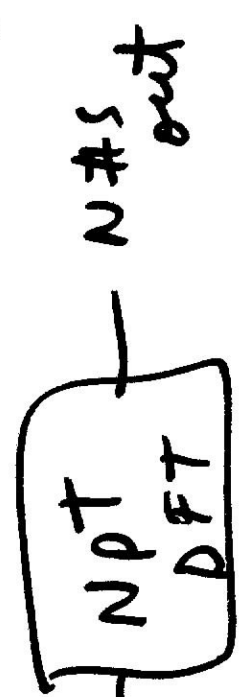
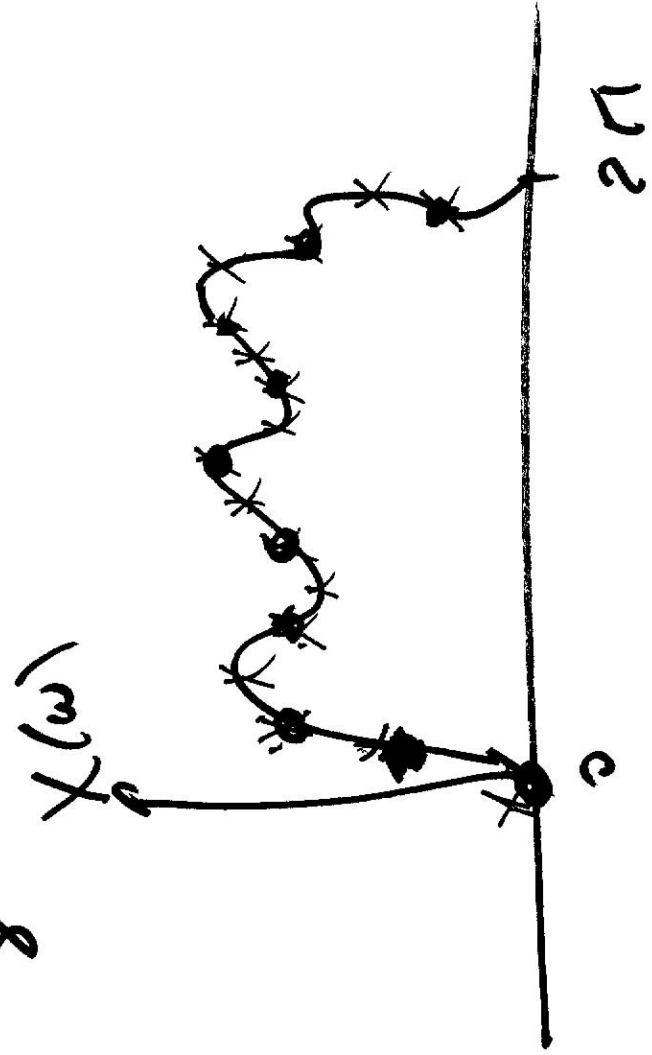
$$1000 \text{ pt DFT of } x(n) : \sum_{n=0}^{999} x(n) e^{-j 2\pi n k / 1000}$$

8 #s

$$8 \text{ pt DFT of } x(n) : \sum_{n=0}^7 x(n) e^{-j 2\pi n k / 8}$$

$x(n)$   $\xrightarrow{\text{MPT}}$   $X(k)$   
 $\text{MPT}$

Pad it  $\rightarrow$   
 $M-N$  zeros



N point seq.  
 padded it N  
 zeros.



Suppose we have nonuniform samples of  $X(\omega)$  at  $N$  points. We know

$X(\omega)$  is DTFT of an N pt seq.

Can I recover  $X(\omega)$ ?  
Linear system of eqns.

Yes solving a linear system of eqns.

$$X(\omega) = \sum_{n=0}^{N-1} x(n) e^{-j\omega n}$$
$$[X(\omega_i)]_{i=0, \dots, N-1} = \sum_{n=0}^{N-1} x(n) e^{-j\omega_i n}$$

$$\begin{bmatrix} X(\omega_0) \\ X(\omega_1) \\ \vdots \\ X(\omega_{N-1}) \end{bmatrix} =$$

$$\begin{bmatrix} e^{-j\omega_0 \cdot 0} & e^{-j\omega_0 \cdot 1} & \dots & e^{-j\omega_0 \cdot (N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j\omega_{N-1} \cdot 0} & e^{-j\omega_{N-1} \cdot 1} & \dots & e^{-j\omega_{N-1} \cdot (N-1)} \end{bmatrix}$$

$N \times N$

Unknown

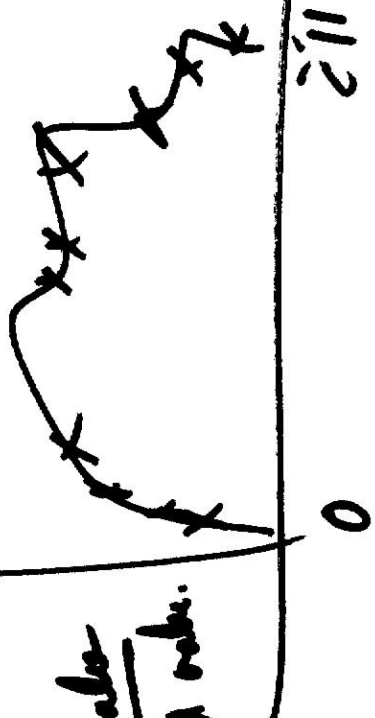
Known:  
non unitary  
square  
NPT  $\rightarrow$   $X$

$A$

NPT.

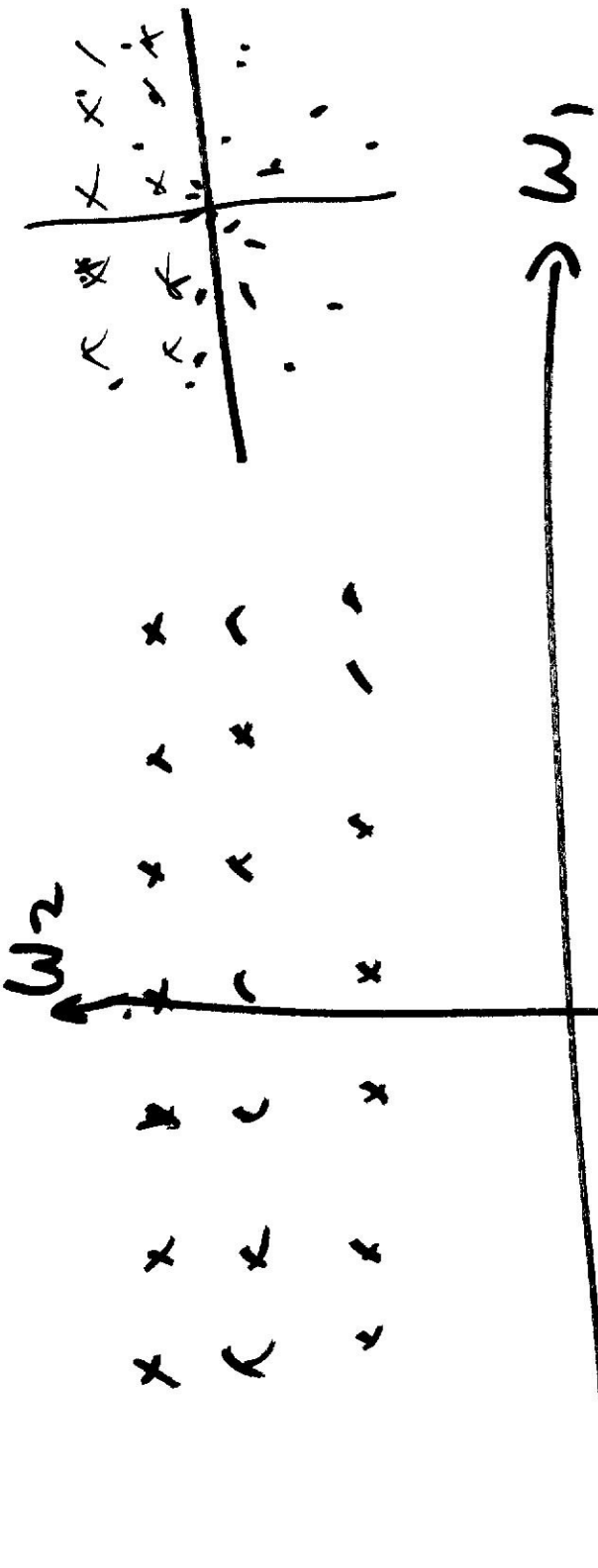
$$Y(\omega) = b$$

$$A \vec{x} = b$$



target signal value  
sampled signal value

Condition  $\neq$



Graph

non-vision sapien in Frey.

1. X Ray Crystallography

widely used  
biology Prog design

2. MRI

Ranker Astromy

3. Radio

U. Tomography

## Thermit Exp:

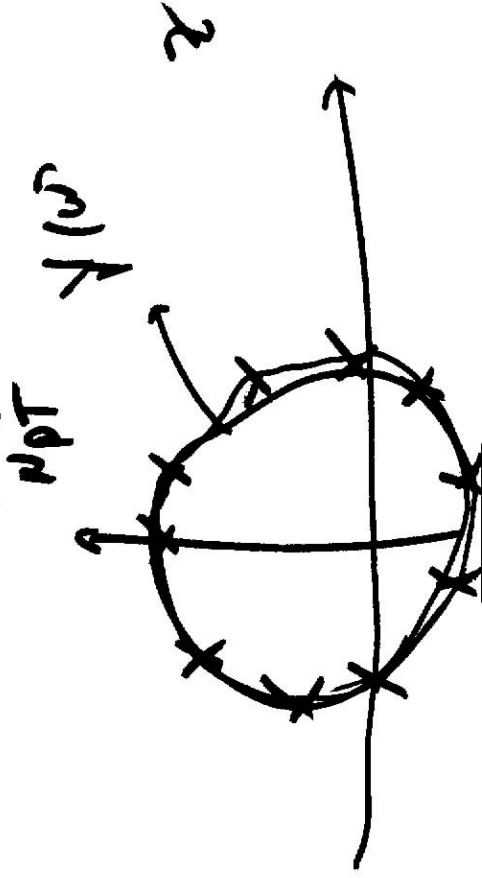
1.  $y(n)$  : either  $\infty$  extent or finite extent.

2.  $Y(\omega) = \text{DTFT} \{ y(n) \}$ .

3. Sample  $Y(\omega)$  at  $N$  equally spaced points.

$$[Y(\omega)]_{\omega = \frac{2\pi k}{N}} = \tilde{X}(k) \quad \swarrow \text{NPT seq.}$$

4. IDTFT  $\{ \tilde{X}(k) \} = x(n) \quad \swarrow \text{NPT seq.}$





Answer: periodize  $y(n)$  with period  $N$ .

$$\tilde{w}(n) = \sum_{k=-\infty}^{+\infty} y(n+kN)$$

extract one period:

$$x(n) = w(n) R_N(n).$$

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$$\text{Proof } x(n) = \begin{cases} \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j 2\pi n k / N} & 0 \leq n < N \end{cases}$$

otherwise

$$\sum_{k=0}^{N-1} \left( \sum_m y(m) e^{-j 2\pi k m / N} \right) e^{j 2\pi n k / N}$$

$$0 \leq n < N$$

otherwise

$$x(n) = \begin{cases} \frac{1}{N} & 0 \end{cases}$$

$$x(n) = \begin{cases} \sum_{m=-\infty}^{+\infty} y(m) \frac{N-m}{N} e^{-j2\pi k(m-n)} & 0 \leq n < N \\ 0 & \text{otherwise} \end{cases}$$

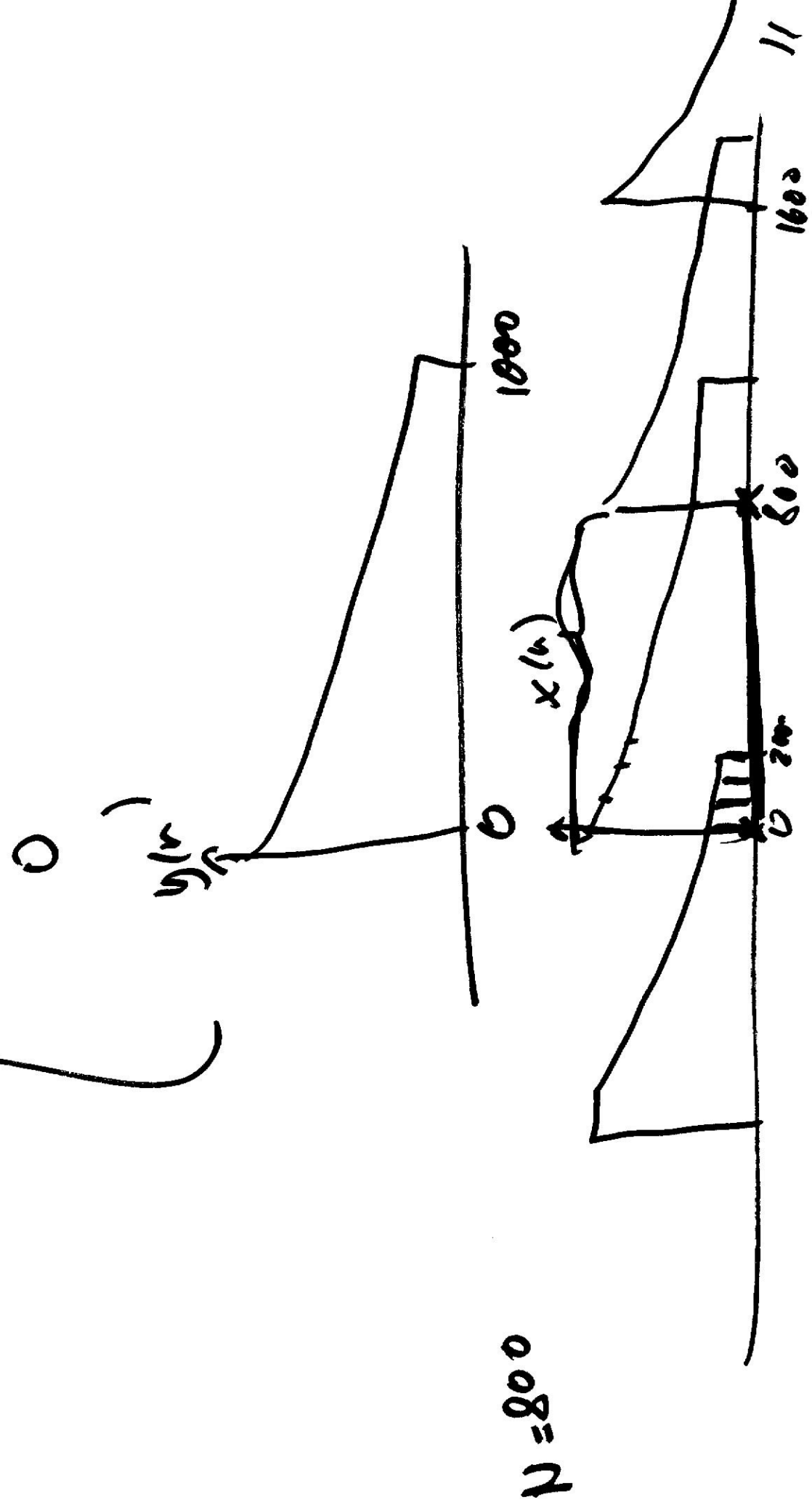
$$= \sum_{r=-\infty}^{+\infty} \delta(n-m+rN)$$

Call  $f(n) = \sum_{r=-\infty}^{+\infty} \delta(n+rN)$

$$x(n) = \sum_{m=-\infty}^{+\infty} y(m) f(n-m)$$

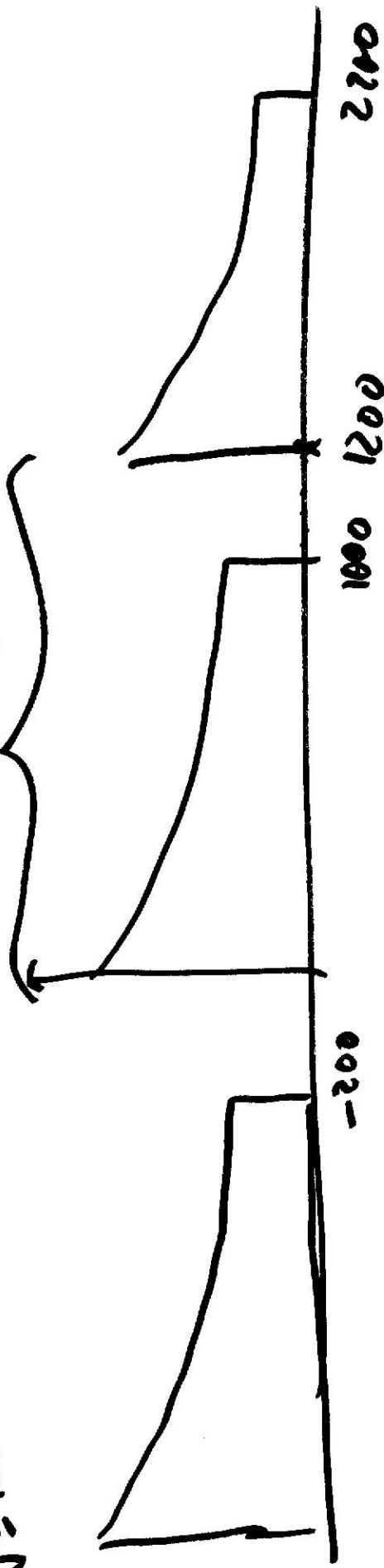
$0 \leq n < N$   
otherwise...

$$x(n) = \begin{cases} \sum_{r=-A}^{+A} y(n+rN) & 0 \leq n < N \\ \text{otherwise} & \end{cases}$$



$x(n)$

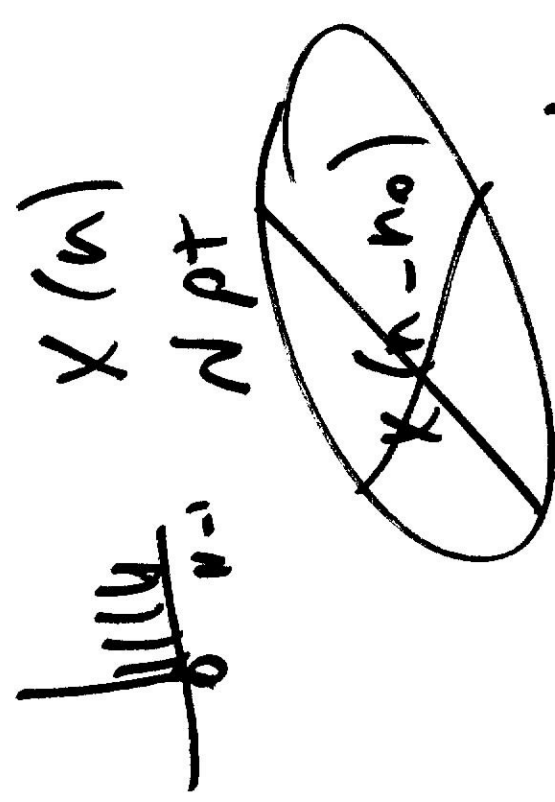
$N=1200$



To get  $|Y(\omega)|$  back precisely, we need  
To sample its DTFT,  $Y(\omega)$  at a higher  
rate than # of samples points in  $g(n)$   
category.

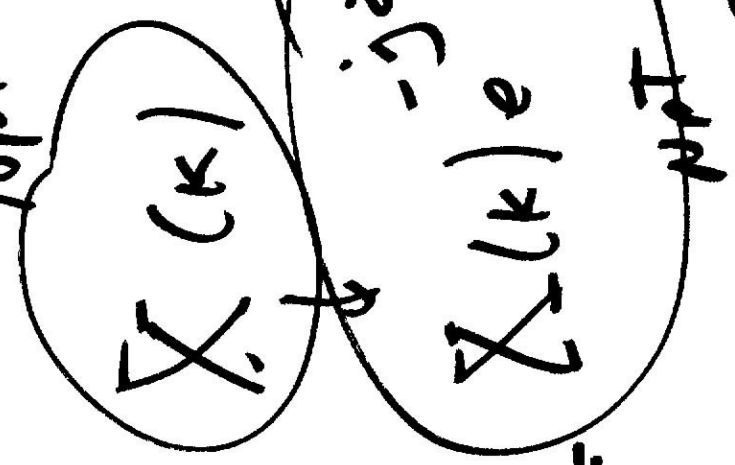
# Properties of DFT

① Shift property:

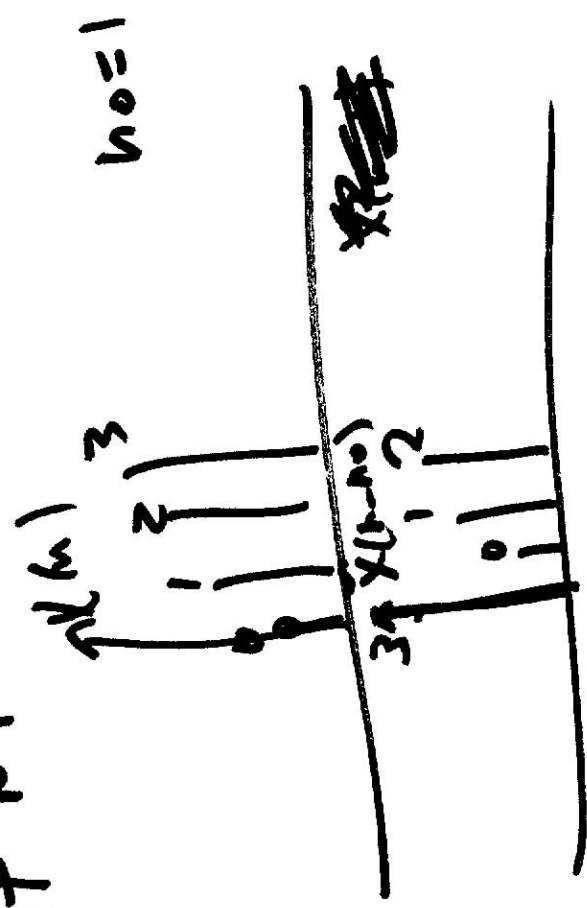


Should only be first  $N$  points  
nonzero for first  $N$  points

$\xrightarrow{\text{DFT}}$



$\xrightarrow{\text{IDFT}}$



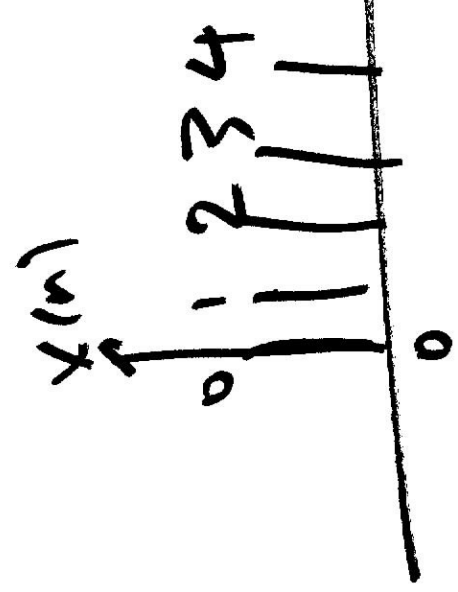
~~Answer~~

$$\tilde{x}(n-n_0) \xleftrightarrow{\text{IDFT}} X(k)e^{-j2\pi n_0 k/N}$$

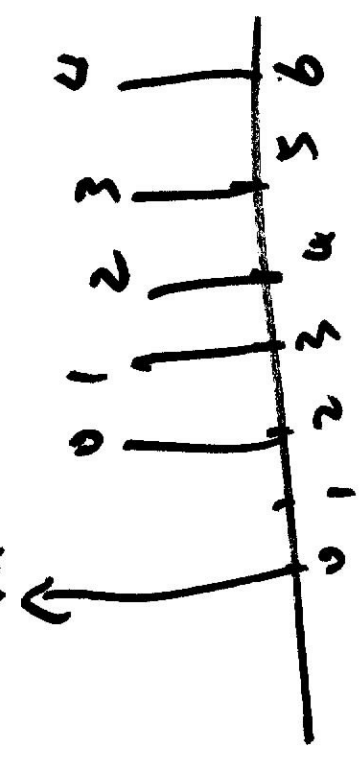
Why?

$$\tilde{x}(n) = \sum_{k=-A}^{+A} X(k+n)$$

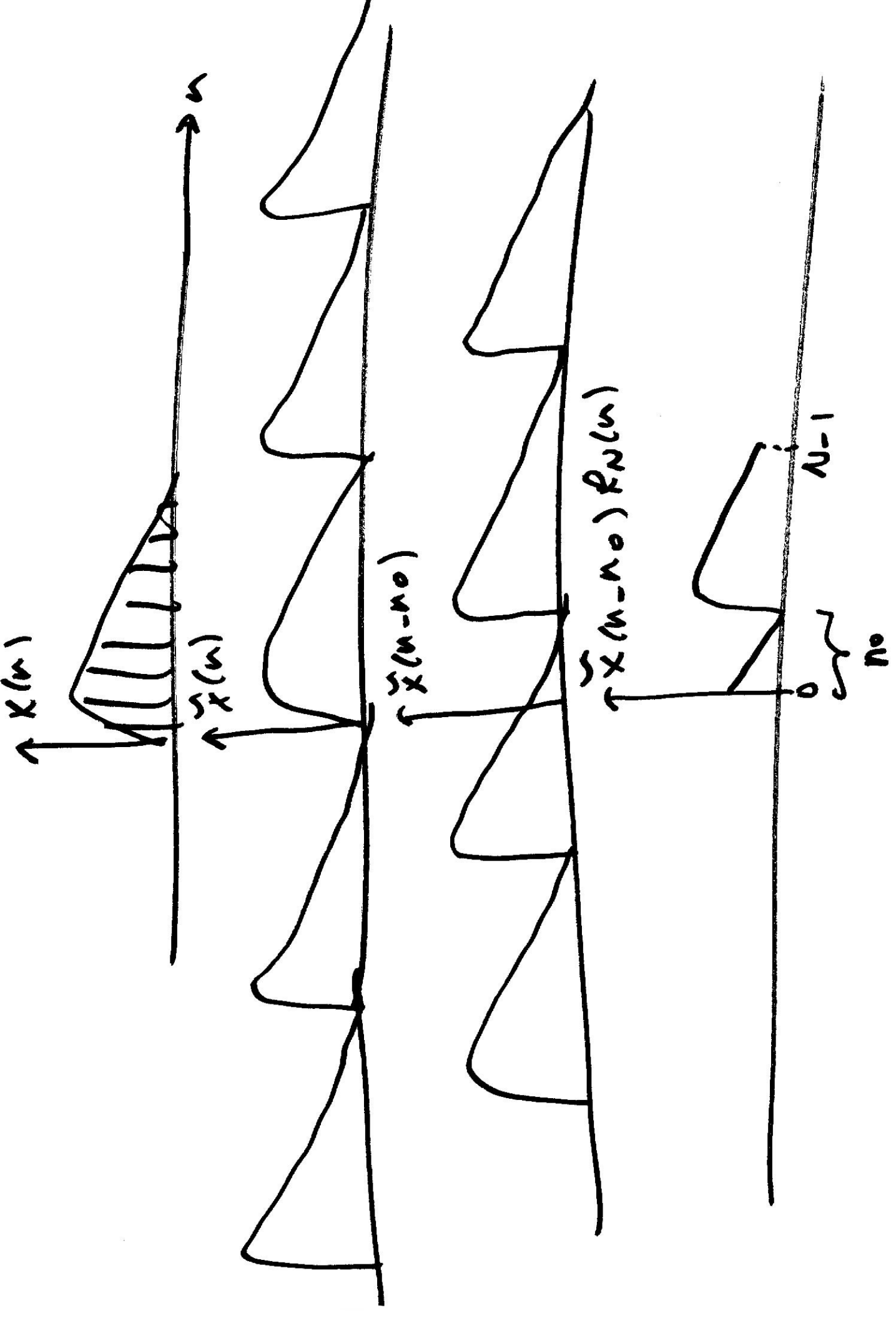
$x(n) \xrightarrow{\text{Periodicity}}$



$x(n-n_0) \quad n=2$

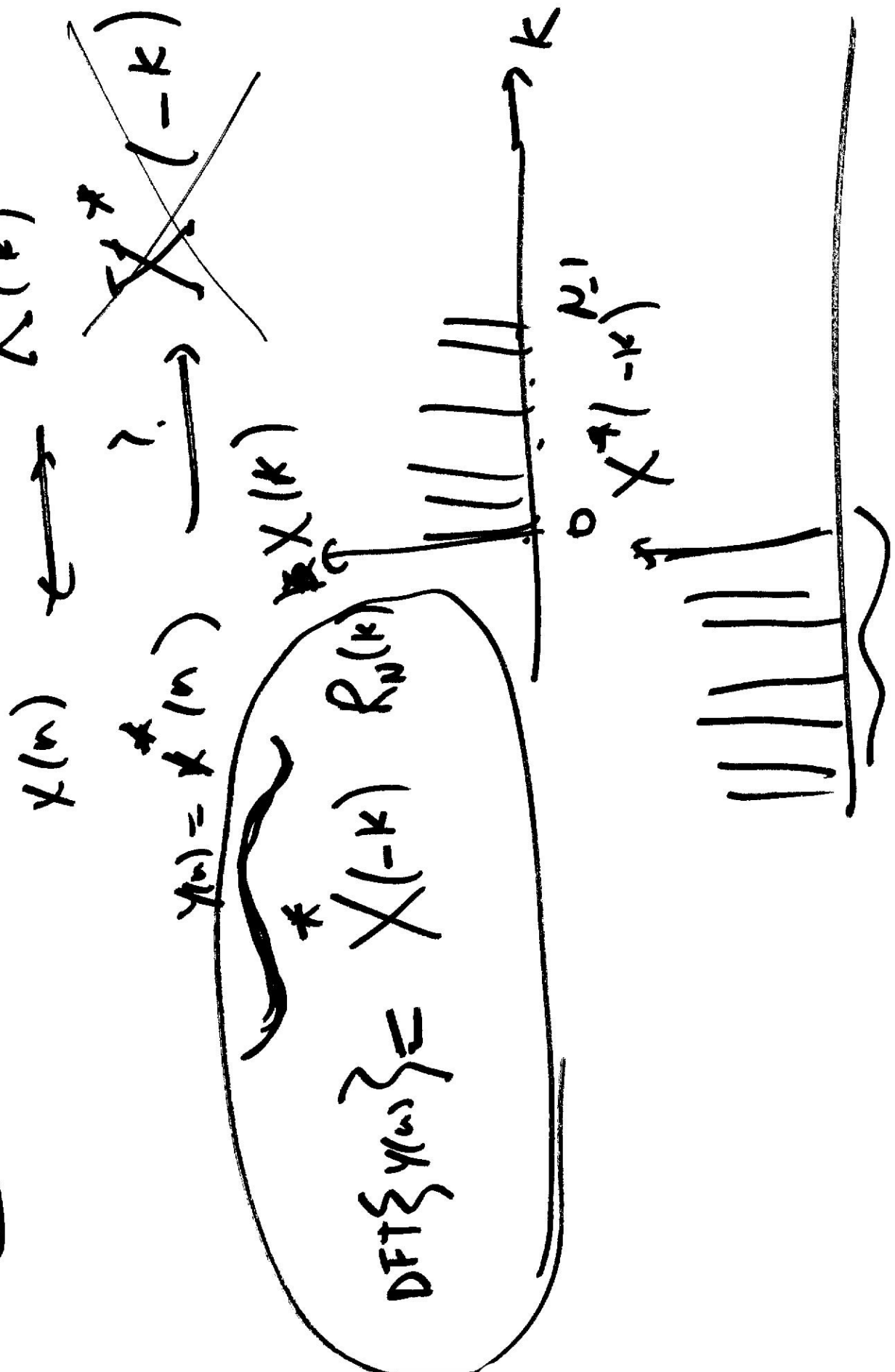


$x(n-n_0)$  is not valid IDFT.



# Another property of DFT:

NFT.





Symmetry Prop

