Realizations of IIR filters with rational transfer function

\[ H(z) = \frac{\sum_{k=0}^{p} b_k z^{-k}}{1 - \sum_{k=1}^{q} a_k z^{-k}} \]

1. Direct Form I
2. \( ^{\top} \)  \( z \)
3. Cascade structure \( \mathcal{G} \)
4. Parallel structure \( \mathcal{G} \)

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Fundamental Theorem of Algebra

Any polynomial in one variable can be factored into simple (1st order) terms.

Polynomial of degree \( n \), has \( n \) real or complex roots. Can be factored into \( n \) terms.

Ex. 3rd order (degree) polynomial.

\[ P(z) = x^3 + \beta x^2 + \gamma x + \delta \]

\[ = K(z - z_0)(z - z_1)(z - z_2) \]

\[ \text{root 1} \quad \text{root 2} \quad \text{root 3} \]
Problem is ill conditioned: if small perturbation in the input results in large change in the output. Well conditioned problem only dependent on the data in the problem itself, not on the algorithm used to solve it. 

$$\begin{bmatrix} 5 & 3 \\ 15 & 8 \end{bmatrix}$$

Ternary solution condition:

$$\begin{bmatrix} 5 & 3 \\ 16 & 6 \end{bmatrix} \approx \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Condition # of a problem: Ratio between Relative change in output over Relative change in input. 

Condition # of problem is large $\implies$ ill conditioned. Small $\implies$ well conditioned.

$$\begin{bmatrix} 2 \\ 15 \end{bmatrix}$$
Problem itself can be inherently either ill-conditioned or well-conditioned. If problem is ill-conditioned, you may...

Condition # of an Alg.

Relative change in output over relative change in the input. Using that alg.

to compute input/output.

If problem is well-conditioned, an ill-conditioned alg. could give bad results.

But a well-conditioned Alg. give good results.
Problem of finding the roots of a polynomial is ill conditioned. The root is worse for polynomials. The root is close to $x_1$, $x_2$, $x_3$ close. Small perturbation in $x_1$, $x_2$, $x_3$ close.
$$H(z) = \frac{P(z)}{Q(z)}$$

$\Rightarrow$ Direct Form 2 is also very sensitive to round off error.

The zeros and poles of the system are very sensitive to round off error.

Finite precision in coefficients affects

Finite precision in calculation.

Motivates: Cancel + Parallel.
Fundamental Thm of Algebra does not hold in 2 or 3 or higher # of variables.

$P(2, z, 2z) = x^2 + 2x^2 + 2z^2 + \beta z^2 + 2z + 2z^2 + 2z^2 + x z^2 + z^2$

The set of factorable polynomials in 2 or more variables is of measure 0.

The set of polynomials in the set of polynomials of measure 0 is of measure 0.

Implication: (1) cannot come up with concordance strongly for 2D JLR.

(2) Cannot come up with concordance strongly for 2D JLR.