Realization of IIR Filters

- IIR
  - IIR with Partial Transfer
    - Direct Form I
    - Direct Form II
  - Cascaded
  - Parallel

- FIR
  - Today
\[ y(n) = \sum_{k=0}^{N} h(k) x(n-k) \]

\[ H(z) = \sum_{k=0}^{N} h(k) z^{-k} \]

\[ y(n) = h(0) x(n) + h(1) x(n-1) + \cdots + h(N) x(n-N) \]

Direct form I, IIR.
Transposed version of

\[ y(n) \rightarrow z^{-1} \rightarrow h(0) \rightarrow z^{-1} \rightarrow h(1) \rightarrow z^{-1} \rightarrow h(2) \rightarrow \cdots \rightarrow z^{-1} \rightarrow h(n-1) \rightarrow z^{-1} \rightarrow h(n) \rightarrow z^{-1} \rightarrow x[n] \]

\[ \text{Cascade} \]

\[ H(z) = \sum_{k=0}^{N} h(k) z^{-k} = \prod_{k=1}^{N} \left( b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2} \right) \]

\[ x[n] \rightarrow b_{01} \rightarrow z^{-1} \rightarrow b_{11} \rightarrow z^{-1} \rightarrow b_{21} \rightarrow \cdots \rightarrow z^{-1} \rightarrow b_{1N} \rightarrow z^{-1} \rightarrow b_{2N} \rightarrow \cdots \]
Many Formats for representation of binary #5.

- ones' complement
- sign & mag.
- two's complement → most commonly used.

- Read # in two's complement with 88 precision.

\[
x = X_m \left( -b_0 + \sum_{i=1}^{8} b_i \cdot 2^{-i} \right)
\]

\[X_m = \text{arbitrary scale factor, } |x| < X_m \]

\[b_i = \text{either zero or } 1\]
\[ b_0 = \text{sign bit} \quad \rightarrow \quad \begin{cases} b_0 = 0 & 0 \leq x \leq x_m \\ b_0 = 1 & -x_m \leq x < 0 \end{cases} \]

With finite number of bits \((B+1)\) we get representation:

\[ x_B = Q_B \lfloor x \rfloor = x_m \left( -b_0 + \sum_{i=1}^{B} b_i 2^{-i} \right) \]

\(x_B\) is the quantized version of \(x\).

\(\Delta\) is the smallest difference between any two in quantized domain: \(\Delta = x_m 2^{-B}\)

5.87923, only use 2 bits: 5.87924
- quantized #s are in the range

\[ X_m \leq x < X_m \]

\[ X_B = b_0 . b_1 b_2 b_3 \ldots b_8 \]

Binary point

5.3924 →

5.4 Rounding

5.3 Truncation
Start a real number \( x \) to get \( x_8 \), one can either

1. \( \text{Truncation:} \quad e = x_8 - x \), where \( e < \frac{1}{2} \) and \( 0 \leq e < \frac{1}{2} \).

2. \( \text{Roundoff:} \quad e = \left\lfloor x \right\rfloor - x \) or \( e = x - \left\lfloor x \right\rfloor \).

Show Fig. 6.37 (a), (b), (c), in QX5.
\[ |x_1| \leq 5000 \]
\[ |x_2| \leq 5000 \]

\[ |x_1 + x_2| < 10000 \]

\[ y = x_1 + x_2 \]

Saturate

Overflow.

Natural overflow

6.38

0.85
Interesting property of two complex:

+ natural overflow:

Add few #s, if the final sum doesn't overflow, then result is correct even though the intermediate results overflow.

Tradeoff between overflow & rounding error.

\[ X_m \uparrow \rightarrow \text{overflow is less likely but } \Delta \uparrow, e \uparrow \]
$X_n \downarrow \quad \text{overflow is more likely.}$

\[\text{but } \mathbf{A} \downarrow \mathbf{e} \downarrow\]

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\[\text{But } B \text{ can come too near.}\]

- Keep $X_n$ large to minimize chances of overflow.

- But keep $B$ large to keep $D$, $e$ small.

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Multiplication also introduces \(\text{overflow}\).