Filter Design

\[ \frac{\text{LTI}}{} \]

FIR
Determining coefficients of \( h(n) \)

IIR

Rational Transfer:
\[ H(z) = \frac{P(z)}{Q(z)} \]

Determining \( P(z) \) and \( Q(z) \)
Implement using D.E.

Not a Rational Transfer
3 steps of building filters

1. Specification → Application dependent
2. Design → Determining coeffs.
3. Realization → Direct form 1, 2, Cascade, Parallel
   - C program on PC
   - Matlab prog.
   - Program DSP chip
   - Xilinx FPGA
   - Programmable ASIC
   - ASIC
FIR Filter Design Using Windows

1. Start with desired freq. Response $H_d(w)$
2. Compute IDFT $\mathcal{F}^{-1} \left\{ H_d(w) \right\} = h_d(n) = \text{desired impulse response}$
3. $h(n) = h_d(n) \cdot W(n)$ \hspace{1cm} \text{finite length window function}
1. High Pass Filter

\[ |H_0(\omega)| \]

Ideal High Pass Filter.

2. Compute I.D.T.F. \( \{ H_0(\omega) \} \)

Assume Linear Phase Filter. (Cosine/ideal, Final FIR)

Assume Type I. \( \beta = 0 \)

\[ d = \frac{N-1}{2} \]

\( \text{if } d \text{ is odd} \)

\[ H_0(\omega) = \frac{H_m(\omega)}{\text{Real}} \]
\[ |H_d(\omega)| = \begin{cases} 
1 & \text{if } \pi - wc < \omega < \pi + wc \\
0 & \text{otherwise}
\end{cases} \]

\[ H_d(\omega) = \begin{cases} 
- \text{j} \omega & \text{if } \pi - wc < \omega < \pi + wc \\
0 & \text{otherwise}
\end{cases} \]

\[ \int_{-\pi}^{\pi} \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} e^{-j\omega n} e^{j\alpha n} \, dn \]

\[ h_d(n) = \frac{(-1)^{n-k}}{\pi (n-k)} \sin \left( \omega_c (n-k) \right) \]

\( \text{Multiply } h_d(n) \text{ by a finite length window to get FIR filter} \)
\[ H(\omega) = \sum h(n) e^{-j\omega n} \]

\[ [H(\omega)]_{\omega = 0} = \sum_{n=0}^{\infty} h(n) \cdot 0.1 \]

\[ \Rightarrow \text{D.C. value of } H(\omega) \text{ is } \sum_{n=0}^{\infty} h(n) e^{-j\omega n} \]

\[ [H(\omega)]_{\omega = \pi} = \sum_{n=-\infty}^{\infty} h(n) e^{-j\pi n} \]

\[ h(n) = \begin{cases} 1.0 & n = 0 \\ -0.1 & n = 1, 2, 3, 4 \\ 0.3 & n = 5 \\ -0.3 & n = 6 \end{cases} \]
\[ h(n) = h_d(n) \times \text{window} \]

\[ H(w)(3) = H(w)(2) \times W(w)(2) \]

\[ W(w)(2) \]

\[ \text{output} \]

\[ \text{input} \]
Terminology

LowPass

\[ H(w) \]

\[ \delta_p = \text{passband ripple} \]

\[ \delta_s = \text{stopband ripple} \]

\[ 0 \leq w \leq W_p \]

\[ W_s \leq w \leq W \]

\[ \Delta W = W_s - W_p = \text{Transition Width} \]
1. Transition width of $H(w)$ depends on mainlobe width of $w$.

2. Ripple in $H(w)$ depends on ripple of window $w$. 

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Q1: How to control transition width $\Delta f$ at $H(w)$, i.e., main lobe of $W(w)$?

$$\Delta f = \frac{4\pi}{M+1}$$

Q2: How to design window $w(n)$ to get good mainlobe/sidelobe behavior $H(w)$?

- Be small
- If small $w_s - w_p$ small

$W(w)$

- Longer windows in time domain
- Have narrower mainlobe width in freq.
- Domain $\Rightarrow$ small transition width in final NR filter
Shape of Window:
Fixed size (duration) window.
but different shapes have different main lobe width.

Fig 7.22 6 vs
Rectangle \rightarrow \text{smallest main lobe width.}
Blackman \rightarrow \text{highest main lobe width.}

Q2 How to design \text{W(n)} to get good side lobe behavior for \text{w(n)} i.e., a good ripple behavior for my final FIR filter.
- Shape of \( \psi(w) \) controls sidelobe behavior.
- But size of \( \psi(w) \) (duration) does not significantly affect sidelobe behavior.

\[ \text{shape} \xrightarrow{\text{sidelobe}} \text{and} \xrightarrow{\text{mainlobe }} \psi(w). \]

\[ \text{size} \xrightarrow{\text{only}} \text{mainlobe of } \psi(w) \]
handle behavior

use size to control behavior of wiki

1. use shape & context stick  
2. track...