Dec 21, 2005

FFT: Decimation in Frequency:

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi nK}{N}} \qquad 0 \le k < N$$

Divide computation 2 parts: even indicies of k $k = 2r$
odd " " " $k = 2r+1$

① k even $k = 2r$

$$X(2r) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi n 2r}{N}} \qquad 0 \le r < N/2$$

$$X(2r) = \sum_{n=0}^{N/2-1} x(n)\, e^{-j\frac{2\pi rn}{N/2}} + \sum_{n=N/2}^{N-1} x(n)\, e^{-j\frac{2\pi rn}{N/2}}$$

$n = m + N/2$

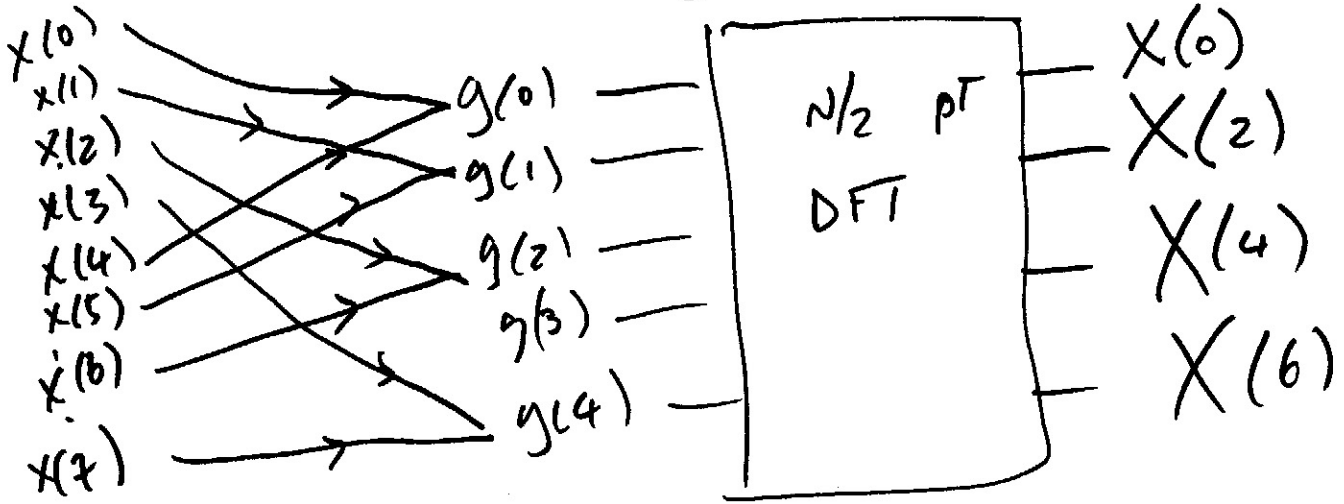
$$X(2r) = \sum_{n=0}^{N/2-1} x(n)\, e^{-j\frac{2\pi rn}{N/2}} + \sum_{m=0}^{N/2-1} x\left(m+\frac{N}{2}\right) e^{-j\frac{2\pi r}{N/2}\left(m+\frac{N}{2}\right)}$$

$$X(2r) = \sum_{n=0}^{N/2-1} x(n)\, e^{-j\frac{2\pi rn}{N/2}} + \sum_{n=0}^{N/2-1} x\left(n+\frac{N}{2}\right) e^{-j\frac{2\pi rn}{N/2}}$$

$$X(2r) = \sum_{n=0}^{N/2-1} \underbrace{\left[x(n)+x\left(n+\frac{N}{2}\right)\right]}_{g(n)} e^{-j\frac{2\pi rn}{N/2}} \qquad 0 \le r < N/2$$

$N/2$ pt DFT of $g(n)$

N=8



Consider $K = 2r+1$ i.e. odd indices of k.

$$X(2r+1) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi n(2r+1)}{N}} \qquad 0 \le r < N/2$$

$$X(2r+1) = \sum_{n=0}^{N/2-1} x(n)\, e^{-j\frac{2\pi n(2r+1)}{N}} + \sum_{n=N/2}^{N-1} x(n)\, e^{-j\frac{2\pi n}{N}(2r+1)}$$

$n = m + N/2$

$$X(2r+1) = \sum_{m=0}^{N/2-1} x\left(m+\frac{N}{2}\right) e^{-j\frac{2\pi}{N}\left(m+\frac{N}{2}\right)(2r+1)} + \sum_{m=0}^{N/2-1} x\left(m+\frac{N}{2}\right) e^{-j\frac{2\pi m}{N}(2r+1)}$$

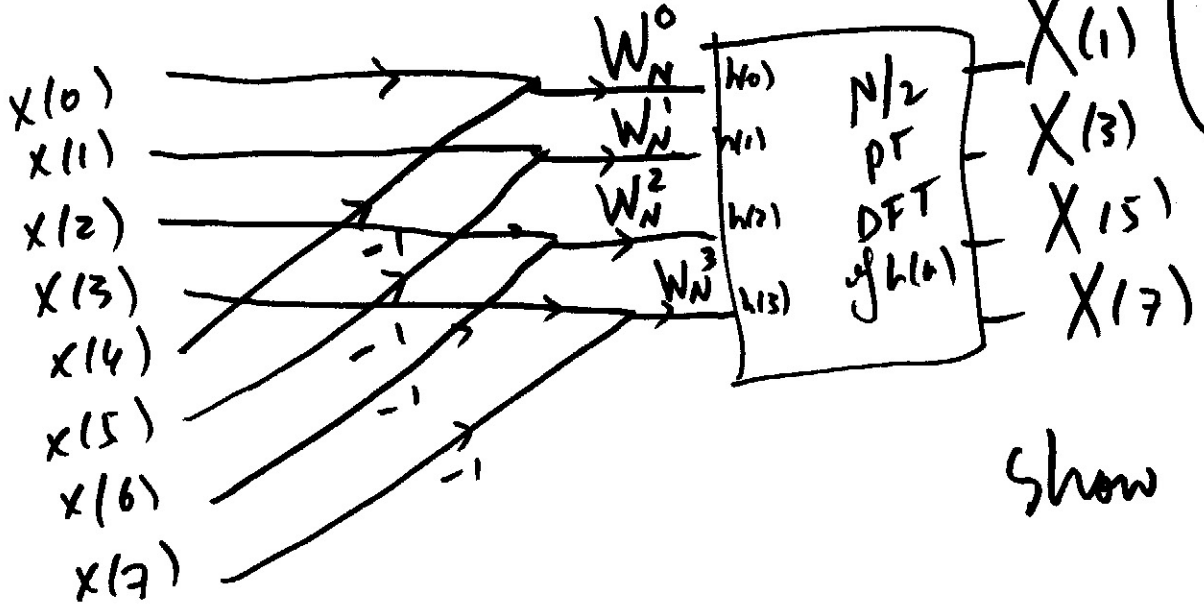
$$X(2r+1) = \sum_{m=0}^{N/2-1} x\left(m+\frac{N}{2}\right) e^{-j\frac{2\pi}{N}\frac{N}{2}(2r+1)} + e^{-1} \sum_{m=0}^{N/2-1} x\left(m+\frac{N}{2}\right) e^{-j\frac{2\pi m}{N}(2r+1)}$$

$$X(2r+1) = \sum_{n=0}^{N/2-1} x(n)\, e^{-j\frac{2\pi n}{N}(2r+1)} - \sum_{n=0}^{N/2-1} x\left(n+\frac{N}{2}\right) e^{-j\frac{2\pi n}{N}(2r+1)}$$

$$X(2r+1) = \sum_{n=0}^{N/2-1} \left[x(n) - x\left(n+\frac{N}{2}\right) \right] e^{-j\frac{2\pi n}{N}} \; e^{-j\frac{2\pi nr}{N/2}}$$

$h(n)$

$N/2$ pt DFT of $h(n) \equiv W_N^n$

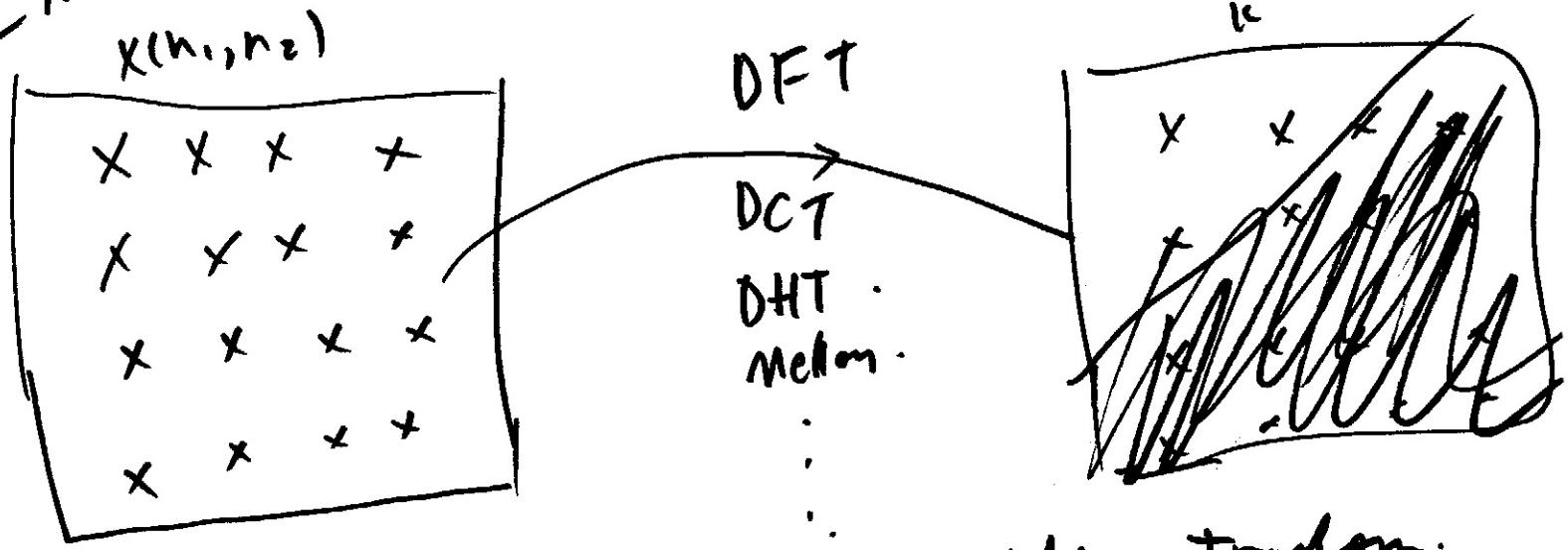


Show 9.17
9.18, 9.19
9.20

Talk about Transposition of Decin Freq → Decin Time

Discrete Cosine Tranfom

$\sum x^2(n)$ $\qquad$ $\sum_k |X(k)|^2$

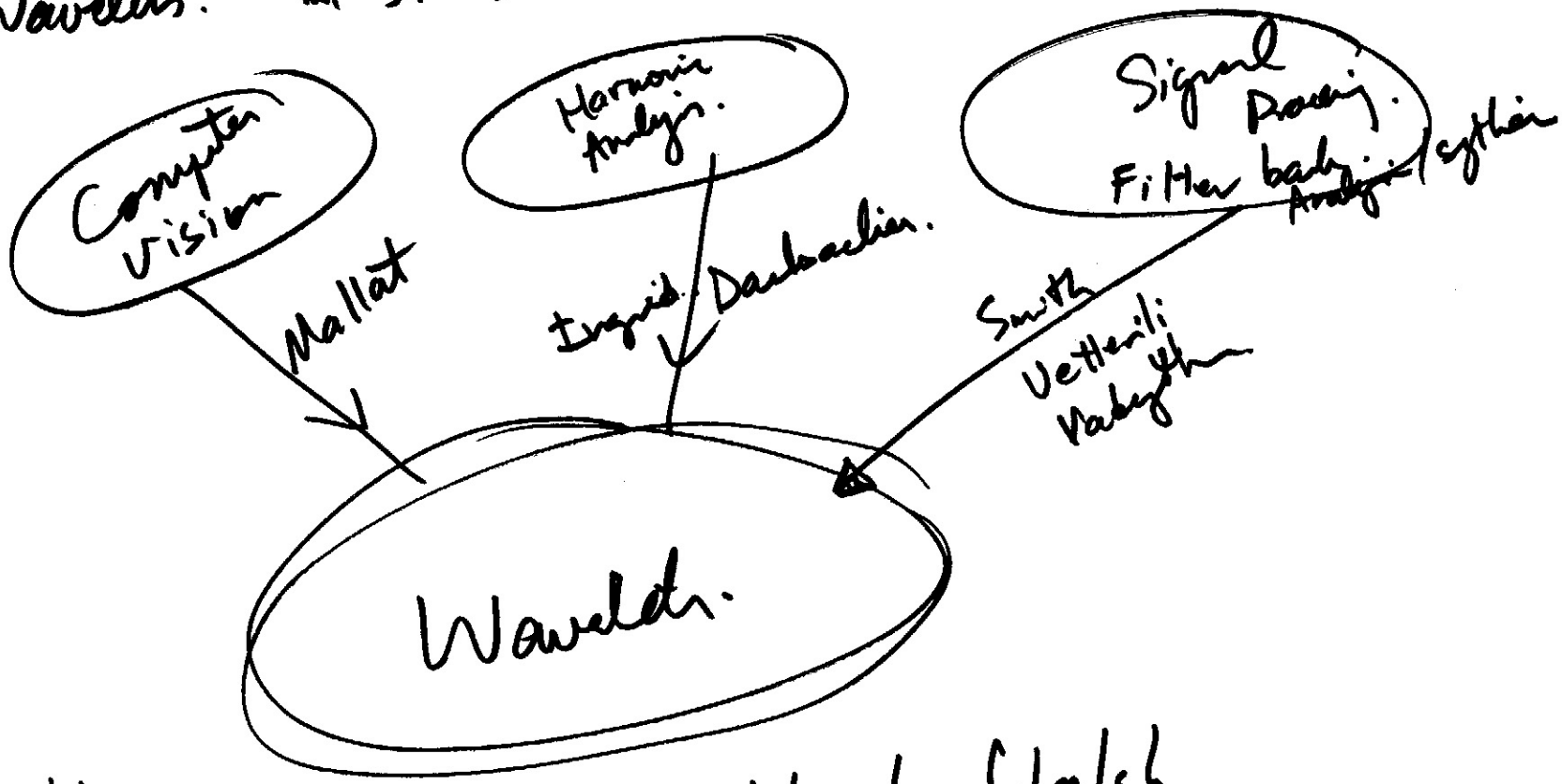


Transform Coding: Basic idea tranfom. the pixels into Tranfom domain, get a bunch of coeffs equal to zero.

- DCT has better "Compaction property". coeffs in Tranfom domain are stat indp.

Used in JPEG, MPEG1, MPEG2, MPEG4
H.261, H.263 H.264

Wavelets: JPEG 2000 ← still image coding

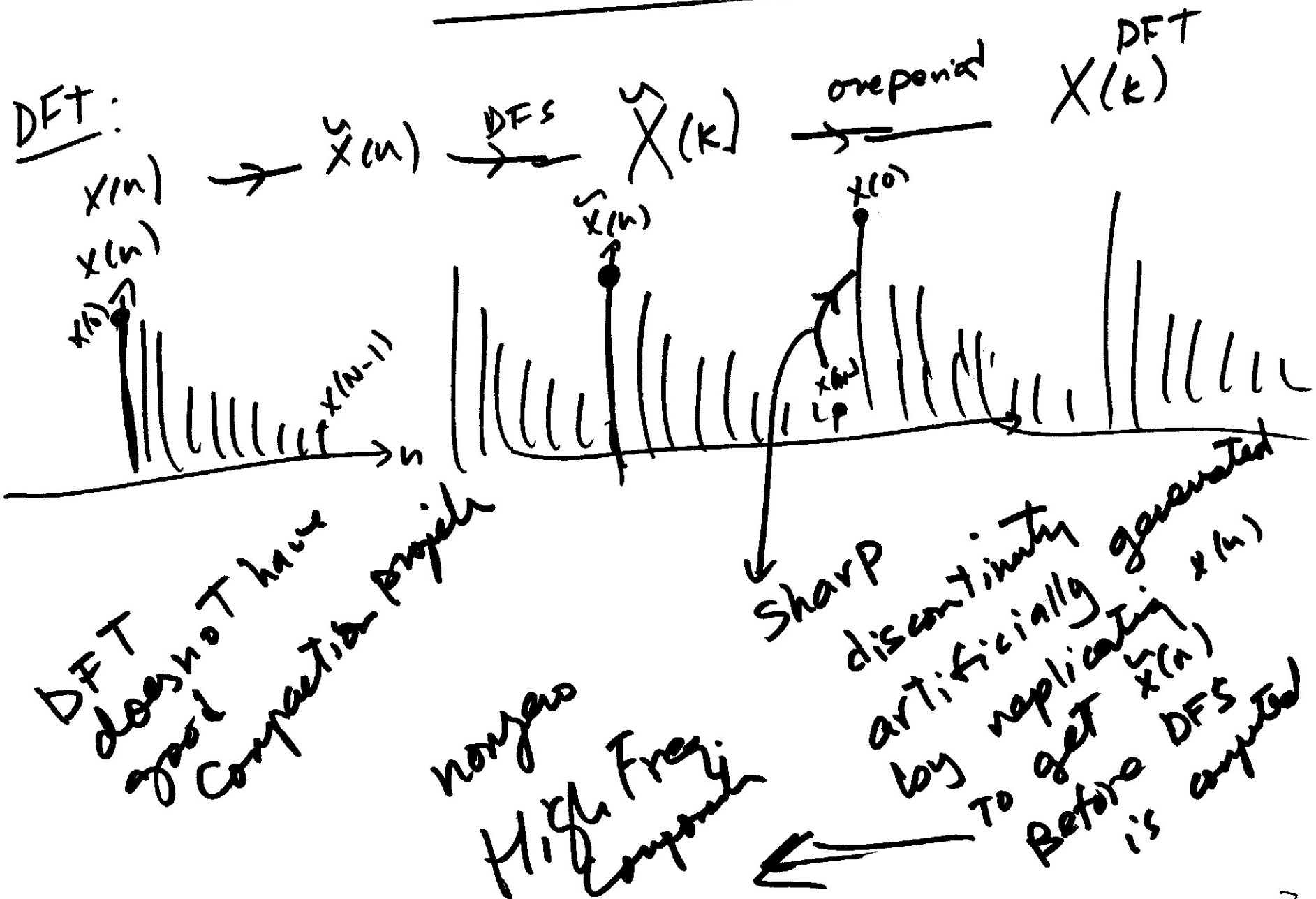


Haar, Hartley, Haddard, Walsh

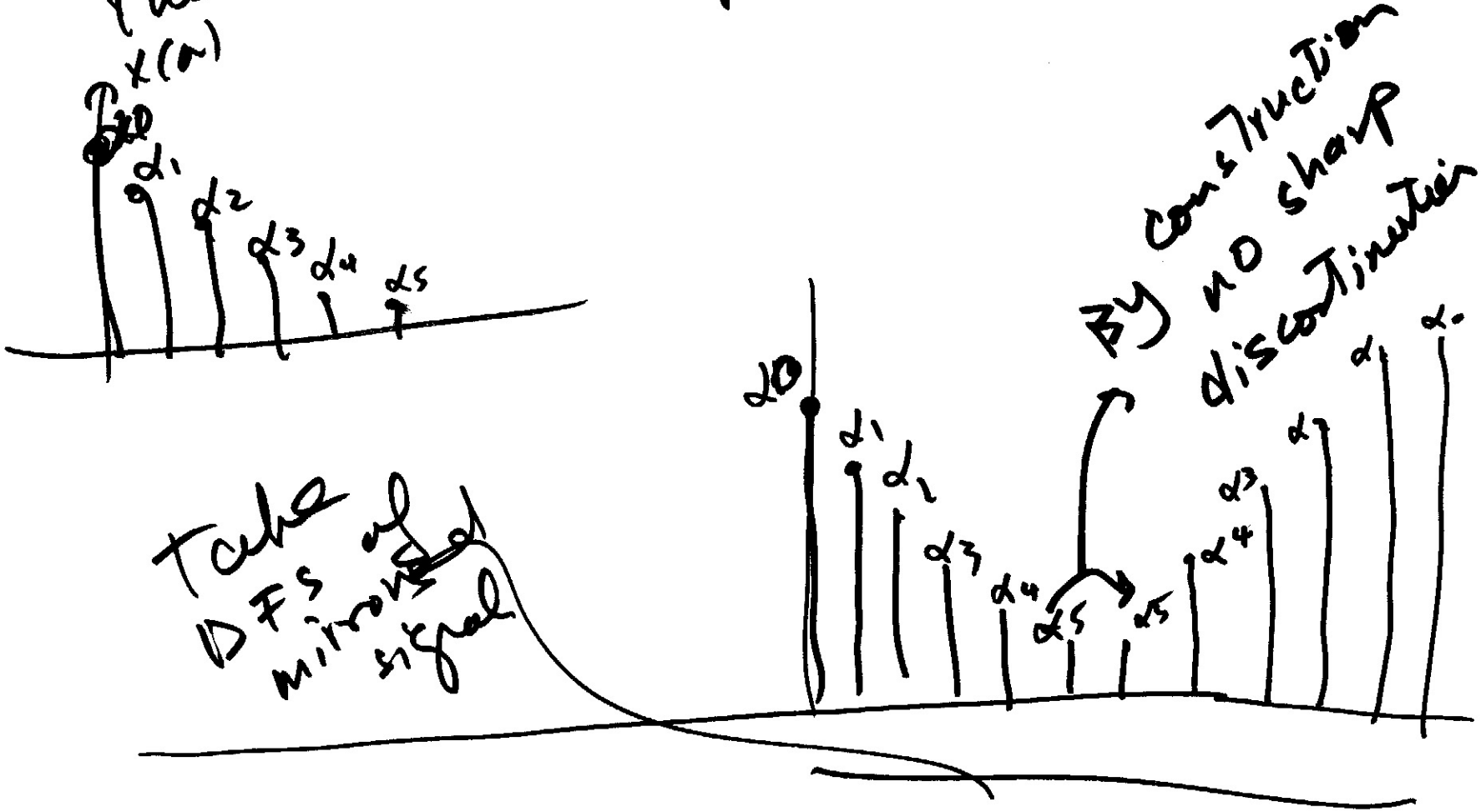
DCT: Real coefficients.
Real arithmetic

DFT: complex coeff.
Complex arithmetic.

Why DCT has better compaction:



Basic idea behind DCT is To replicate signal in a "better" way so that there is no sharp discontinuities



4 Kinds of DCT

We'll focus on DCT 2

$$X_{C_2}(k) = 2 \sum_{n=0}^{N-1} x(n) \cos\left(\frac{\pi k (2n+1)}{2N}\right)$$

analysis

Synthesis:

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} \beta(k) X_{C_2}(k) \cos\left(\frac{\pi k (2n+1)}{2N}\right)$$

$$\beta(k) = \begin{cases} \frac{1}{2} & k = 0 \\ 1 & 1 \le k < N \end{cases}$$

Relation Between DFT and DCT 2

Proposal ①:

① $x(n)$: N pt real seq.

② 2N pt DFT $\longrightarrow$ $X(k)$

③ $2\,\mathrm{Re}\left\{ X(k)\, e^{-j\frac{2\pi k}{2N}} \right\} \longrightarrow$ DCT-2

Proposal ②

① start with N pt real seq. $x(n)$

② Pad it with N zeros $\longrightarrow$ $x_{2N}(n)$

③ Form a periodic seq.

$$\hat{x}_2(n) = \hat{x}_{2N}(n) + \check{x}_{2N}(-n-1)$$

④ take 2N pt DFT of one period of $\hat{x}_2(n)$

$$X_2(k)$$