Rolloff Noise issues in Cascade Structure

Spec: \[ 0.99 \leq |H(\omega)| \leq 1.01 \quad |\omega| < 0.5\pi \]
\[ |H(\omega)| \leq 0.01 \quad 0.5\pi \leq |\omega| < \pi \]

\[ H(z) = 0.0794 \quad \prod_{k=1}^{3} H_k(z) = 0.0794 \prod_{k=1}^{3} \frac{(1 + b_{1k}^{-1}z^{-1} + b_{2k}^{-2}z^{-2})}{(1 - a_{1k}^{-1} - a_{2k}^{-2}z^2)} \]

<table>
<thead>
<tr>
<th>( K )</th>
<th>( a_{1k} )</th>
<th>( a_{2k} )</th>
<th>( b_{1k} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.47</td>
<td>-0.17</td>
<td>1.7</td>
</tr>
<tr>
<td>2</td>
<td>0.14</td>
<td>-0.6</td>
<td>.78</td>
</tr>
<tr>
<td>3</td>
<td>-0.05</td>
<td>-0.90</td>
<td>.411</td>
</tr>
</tbody>
</table>

Unity gain at passband
Can show $0.0744$ prevent overflow in the above implementation.

Add quantization noise source

$$H(z) = S, H_1(z) = s_1, H_2(z) = s_2, H_3(z) = s_3$$

Choose $S, S_1, S_2, S_3$ so that no overflows in $H_1, H_2, H_3$
If you apply Technique #2 from last lecture.

\[
S_0 \times_{max} < \frac{1}{\max_{k, \omega} |H_k(\omega)|} \Rightarrow
\]

\[
S_1 \Rightarrow \text{only worry in terminal nodes of } H_1
\]

\[
S_2 \Rightarrow \text{only consider internal nodes of } H_2
\]

\[
S, S_2 \leq S_3 = 0.00794
\]

\[
S_1 = 0.186 \quad S_2 = 0.52 \quad S_3 = 0.80
\]
\[ S_1, \max | H_{im}(w) | < 1 \]
\[ |\omega| < H, m \]

\[ H_{im}(w) \text{ are transfer fn between input to } H_1 \text{ and all of its node, } m. \]

\[ S_1, S_2, \max | H_{im}(w) H_{2n}(w) | < 1 \]

\[ S_1, S_2, S_3 = 0.794 \]

\[ S_1, S_2, S_3, \text{ there exists their inverse} \]

\[ \text{Their inverses} \]
noise variance at output
\[ \begin{align*}
\text{noise variance at output} & = 5 \frac{2B}{12} \left[ + \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{s_2^2 |H_3(\omega)|^2}{|A_2(\omega)|^2} d\omega \\
&+ \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{s_3^2 |H_3(\omega)|^2}{|A_3(\omega)|^2} d\omega \\
&+ \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{s_2^2 |H_2(\omega)|^2}{|A_1(\omega)|^2} d\omega \\
&+ \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{|A_1(\omega)|^2} d\omega \right] 
\end{align*} \]

Observation: 
- $H_1$ does not appear
- $H_2$ appears 1
- $H_3$ appears 2

\[ \rightarrow \text{lots of different 6s depending on how we pair up zeros + poles.} \]
Jackson's rule for choosing a zero/pole pair:

1. Pair up a pole that is closest to the unit circle with the zero that is also closest to the unit circle.
2. Repeat rule 1 until all zeros and poles are exhausted.
3. The resulting 2nd order section should be ordered according to either increasing closeness to the unit circle or decreasing closeness to the unit circle.
noise power \( n(t) \rightarrow \text{stationary} \)

Auto correlation

\[ A(\tau) = E[n(t)n(t+\tau)] \]

F.T.

Power spectrum of noise

\[ EE126 \]

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Fixed Point $\leftarrow$ int
\[ \rightarrow \] long
\[ \rightarrow \] Short

Flooring Point

Characteristic
Exponent
\[ f = 2^{\hat{c}} \]

\[ C, x_m \]

Fixed point

\[ 0.5 < x_m < 1 \]
Floating point arith

1. no over-flows
   Addition

2. no multiple
   multipliction

Both error are signal

dependent

\[ x = x(1+e) = x + e \times x \]