Thought Exp

- Correct way: Pad both seq with enough zeros to get \( L + P - 1 \) pt sequence.
  - Multiply DFT of these \( L + P - 1 \) pt seq.
  - Take IDFT of the product.

- Wrony Way
  - Suppose take \( L \) pt DFT of \( x_1 \) and \( x_2 \).
  - Multiply two \( L \) pt DFT
  - Take \( L \) pt IDFT of the product.
Claim: Only the first $P$ points are correct.

The answer obtained via the "wrong way" is really wrong. The rest are good.

First $P-1$ are wrong. Remaining point up to index $L-1$ on $L$th output is right.
Wrong Way

$L$ pt

Answer at location 1 is off.

Answer at location 2 is also off.

Correct
Overlap Save

1. Segment sequence into L point chunks, overlapping with each other by P-1 points.

2. L pt circular convolution of each chunk. Multiply L pt DFT of chunk by L pt DFT of X2 \[ \rightarrow \] IDFT \[ \rightarrow \] L pt.

3. Throw away the first P-1 point of the IDFT in part 2, replace it with answer obtained from previous segment.
Fast Fourier Transform

Decimation in Time

\[ X(k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi nk}{N}} \]

For each \( k \rightarrow N \) adds

\( N \) value of \( k \rightarrow N^2 \) adds

Direct way of computing DFT.

\( O(N^2) \)
Decimation in time

\[
X(k) = \sum_{n \text{ even}} \sum_{0 \to N-1} x(n) e^{-j\frac{2\pi nk}{N}} + \sum_{n \text{ odd}} \sum_{0 \to N-1} x(n) e^{-j\frac{2\pi nk}{N}}
\]

\[
x(k) = \sum_{r=0}^{N/2-1} x(2r) e^{-j\frac{2\pi kr}{N}} + \sum_{r=0}^{N/2-1} x(2r+1) e^{-j\frac{2\pi (2r+1)k}{N}}
\]

\[
x(k) = \sum_{r=0}^{N/2-1} g(r) e^{-j\frac{2\pi kr}{N/2}} + \sum_{r=0}^{N/2-1} h(r) e^{-j\frac{2\pi k r}{N/2}}
\]

\[
x(k) = \text{N/2 pt DFT of } g(r)
\]

\[
x(k) = \text{N/2 pt DFT of } h(r)
\]
\[ G\left( \frac{N}{2} \right) = G(0) \]
\[ G\left( \frac{N}{2} + 1 \right) = G(1) \]
\[ G\left( \frac{N}{2} + k \right) = G(k) \]
\[ X(n) = G(n) + \frac{1}{16} e^{j\frac{2\pi n}{16}} H(n) \]

\[ G(n) = G(3), \quad H(n) = H(3) \]
Repeat the above process until I get to a 2 pt DFT.

\[ \sum_{n=0}^{1} x(n) e^{-j2\pi nk/2} = x(0)e^{-j\pi k} + x(1)e^{-j\pi k} \]

\[ = x(0) + x(1)e^{-j\pi k} \]

\[ k = 0 : e^{-j\pi k} = 1 \Rightarrow X(0) = x(0) + x(1) \]

\[ k = 1 : e^{-j\pi k} = -1 \Rightarrow X(1) = x(0) - x(1) \]
New Notation

\[ W_N^k \triangleq e^{-j2\pi k / N} \]

Twiddle factor.

Use twiddle factor to redraw my basic flow graph for Dec. in Time.

Fig 9.3 of 025.

9.7 of 025.