Effects of round-off noise in Digital Filters

- Direct form IIR.

\[ y(n) = \sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \]

Direct form I:

2nd order system

Assume signal values & coefficients are represented by \((B+1)\) bit fixed point binary numbers.
When multiply $2$ CEs that are each $(B+1)$ bits, reduce to $(2B+1)$ bits and quantize to $(B+1)$ bits.

\[ y(n) = \hat{y}(n) \]

\[ Q \left[ b \times (m) \right] - b \times (n) = e(n) \]
Assumption: ① \(e(n)\) wide sense stationary white noise process.

② Each quantizer same has uniform distribution of amplitude over one quantization interval.

③ Each quantization noise source is uncorrelated with its input, all other quantization noise source and input to system \(x(n)\).
last time lect.

\[-\frac{1}{2} 2^{-B} \leq e(m) \leq \frac{1}{2} 2^{-B}\]

Truncation

\[-2^{-B} \leq e(n) \leq 0\]

Redraw, mean value of \( e(n) \) =

Assume uniform dist.

\[m_e = 0 \quad \sigma_e = \frac{2}{12}\]

\[\Delta X = \frac{1}{2} 2^{-B}\]
Can show $e(n) = e_0(n) + e_1(n) + e_2(n) + e_3(n) + e_4(n)$.

Then can rewrite $E[m]$.
Invoke Assumption 3

\[
\text{Var}[e(n)] = \sigma_e^2 = \sigma_{e_0}^2 + \sigma_{e_1}^2 + \sigma_{e_2}^2 + \sigma_{e_3}^2 + \sigma_{e_4}^2 \\
= 5 \sigma_{e_i}^2 = 5 \frac{2}{12} - 2B
\]

(2nd) order scab.
\[ f(m) = \sum_{k=1}^{L} e_k \Rightarrow \text{mean of } L \text{ is } \frac{1}{L} \sum_{k=1}^{L} e_k = \text{me}_L \text{ of } L \text{ is } e_L \]

As 

\[ e \text{ is w.s.s. to mean of } f \]

\[ f = \text{me}_L \text{ of } e \]

\[ e = \text{mean of } f \]

\[ \text{Hess} \Rightarrow \text{mean of } f \text{ is } \text{me}_L \text{ of } L \text{ is } e \]
\[ G_f^2 = \frac{G_0^2}{\sum_{n=-\infty}^{+\infty} |h_{ef}(n)|^2} \]

\[ h_{ef}(z) = \frac{1}{A(z)} \]

\[ G_f^2 = \left( M + (\pm n) \right) \frac{Z}{12} \sum_{n=-\infty}^{+\infty} |h_{ef}(n)|^2 \]

\[ Z = 2^{\frac{2B}{2}} \sum_{n=-\infty}^{+\infty} |h_{ef}(n)|^2 \]
First order system

\[ H(z) = \frac{b}{1 - a_1 z^{-1}} \]

\[ e(n) = e(n+b) + e(n-b) \]

\[ x(n) \quad y(n) \]

\[ a = \frac{1}{2} \quad b = \frac{1}{2} \quad c_f = 2 \]

\[ z = \frac{-2b}{2} \quad \frac{1}{2} \quad \frac{-2b}{2} \quad \frac{1}{2} \quad 1 \]

\[ n = 0 \]

\[ 1 \quad 2 \quad 3 \]

\[ 19 \quad 19 \quad 19 \]

\[ 19 \quad 19 \quad 19 \]
\[ E(\xi^2) = \mathbf{H}_1(\xi^2) = \int_{-1}^{+1} \frac{\sin^2 \theta}{(1 - r^2)(1 - \text{re}^{i\theta})^2} \, \text{d}w \]
Direct form 2:
Direct form II

\[ b^2 = N \left( \frac{2}{12} \sum_{n=-B}^{+B} |h(n)|^2 \right) \]

\[ + (M+1) \left( \frac{-2B}{12} \right) \]

\[ B \rightarrow 16 \rightarrow 3 \text{sec} \]

\[ N \rightarrow a \rightarrow \text{polin} \]