OCTOBER 31st

↳ Midterm II November 14th

↳ Assign project on Nov 7th

Today: a few words on multirate

↳ using compressors and expanders can yield savings in computation

4.7.1: interchange filtering with compressor/expander

compressor:

claim

$$x[n] \to \boxed{\downarrow M} \to \boxed{H(z)} \to y[n] \equiv x[n] \to \boxed{H(z^M)} \to \boxed{\downarrow M} \to \tilde{y}[n]$$

$\triangleq v[n]$

$V(e^{j\omega}) = H(e^{j\omega M}) \cdot X(e^{j\omega})$

Proof

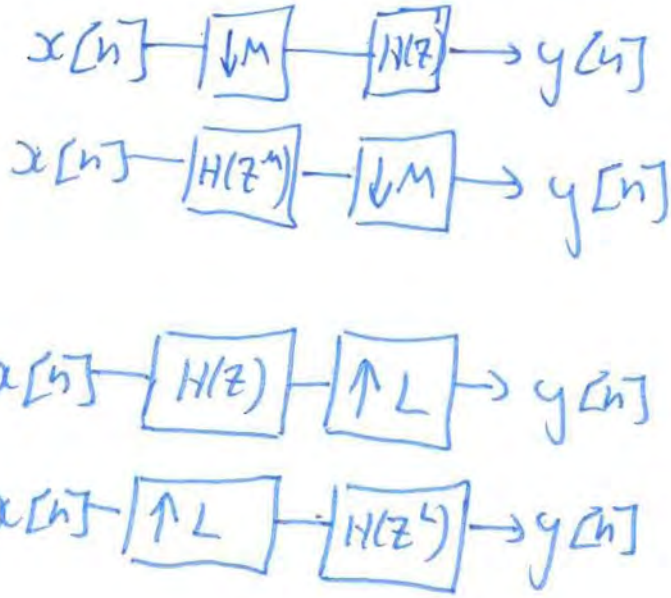
$$Y(e^{j\omega}) = H(e^{j\omega})\left(\frac{1}{M}\sum_{i=0}^{M-1} X\left(j\left(\frac{\omega}{M} - \frac{2\pi}{M} i\right)\right)\right)$$

$$= \frac{1}{M}\sum_{i=0}^{M-1} \underbrace{H(e^{j(\omega - 2\pi i)})}_{= H(e^{j\omega})} X\left(e^{j\left(\frac{\omega}{M} - \frac{2\pi i}{M}\right)}\right)$$

$$= \frac{1}{M}\sum_{i=0}^{M-1} H\left(e^{jM\left(\frac{\omega}{M} - \frac{2\pi i}{M}\right)}\right) X\left(e^{j\left(\frac{\omega}{M} - \frac{2\pi i}{M}\right)}\right)$$

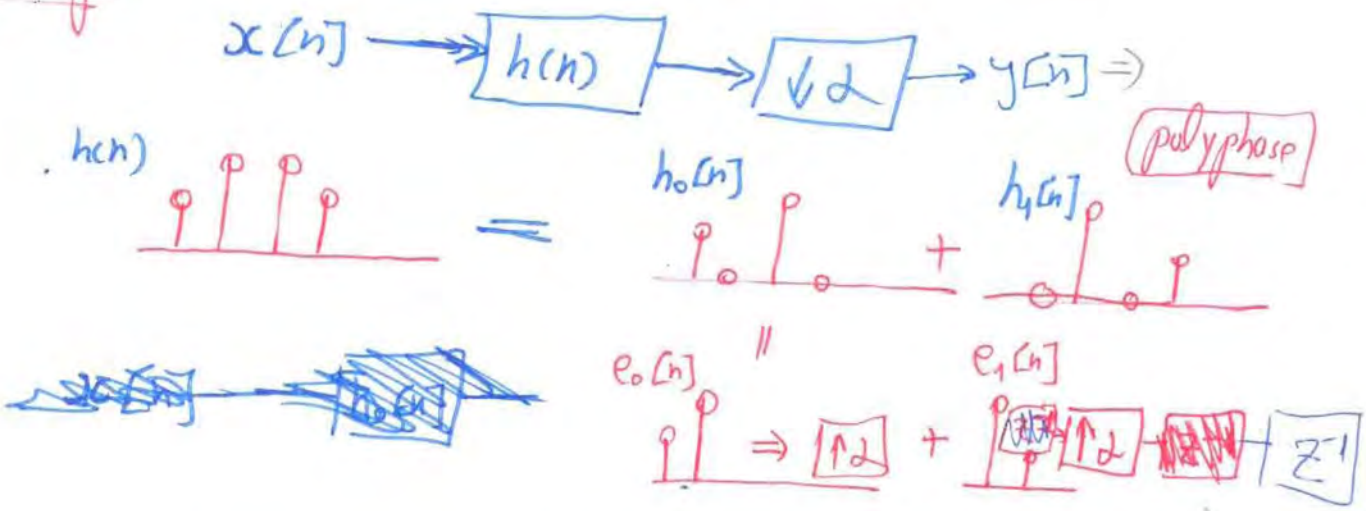
$$= \frac{1}{M}\sum_{i=0}^{M-1} \underbrace{V\left(e^{j\left(\frac{\omega}{M} - \frac{2\pi i}{M}\right)}\right)}_{\text{result of compressor after } V[n]} = \tilde{Y}(e^{j\omega})$$

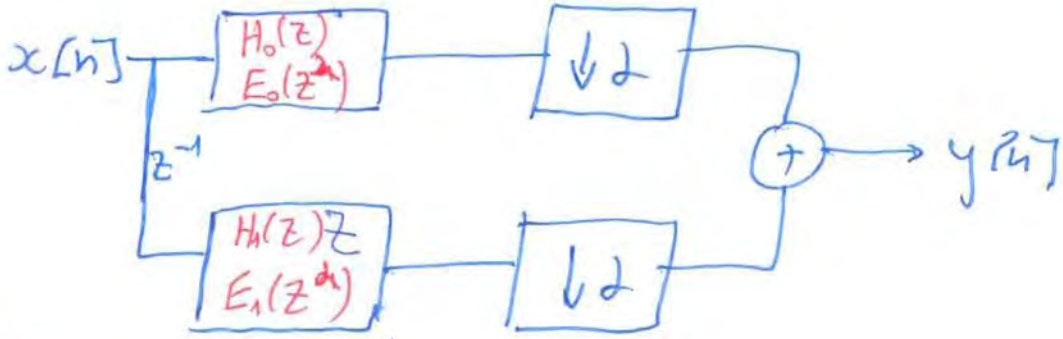
Sumary:



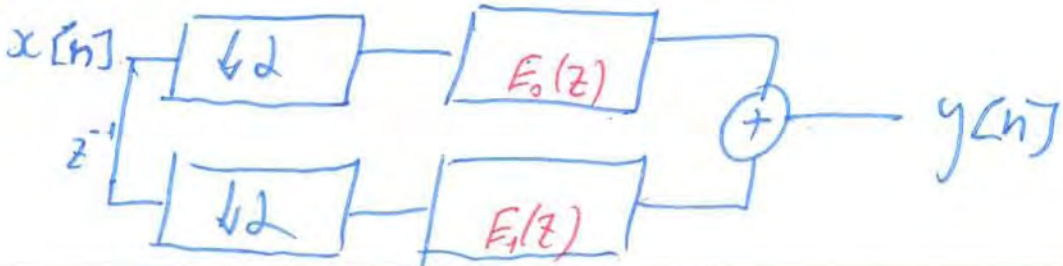
why useful?

Example:





or:



<u>complexity:</u>

(1) streightforeword 4 mult / clock

(2) polyphase → (half clock rate * half mult / filter) ×
× two filters $= 4 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 2 = \frac{4}{2} = 2$ mult / clock

multi-rate appears in filter-banks → wavelets!

CH. 7: Filter design Techniques

Types of D.T Filters:

Infinite impulse response (IIR)

- Impulse response has infinite duration
- Implemeted with recursion (feedback)

Pros

- fewer delays and multiplies for given performance

Cons

- more susceptible to errors and instability due to finite word length.
- Generally not linear phase (different delays for each frequency)

Finite Impulse Response (FIR)

- Response has finite duration
- No recursion

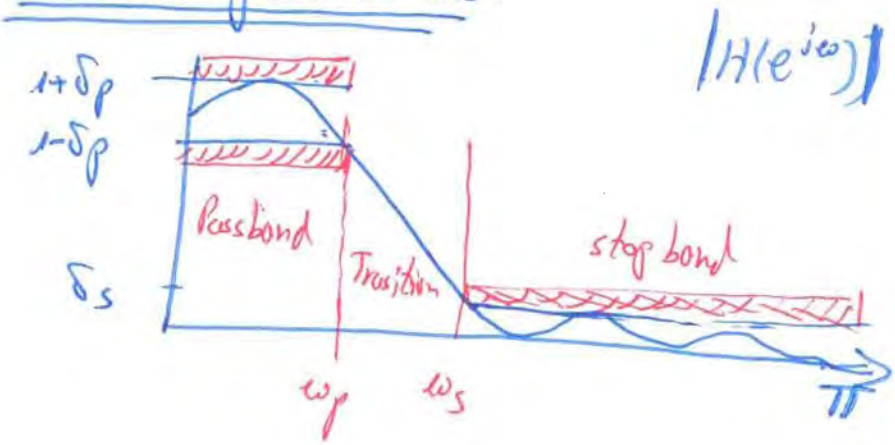
Pros

- Alway stable! (poles only at $z=0$)
- Less susceptible to quantization
- Can design linear phase
- ~~Easy~~ Can to design arbitrary filters.

Cons:

- More delays and multiplies → not a problem anymore
- Design can be more difficult ∴ not a problem

Filter specifications:



Specs:

$\omega_p \rightarrow$ Pass band edge

$\omega_s \rightarrow$ stop band edge

$\delta_p \rightarrow$ peak band-pass ripple

$\delta_s \rightarrow$ peak stop-band ripple

Ideal: $\delta_p = \delta_s = 0$, $\omega_p = \omega_s \Longrightarrow$ not realizable.

IIR Design

Historically → Continuouse IIR design was highly advanced use results from C.T to D.T

- C.T IIR designs have closed form → easy to use.
- With these designs, easy to control amp. response but not phase response.

Common types:

(1) Butterworth → monotonic, no ripple

(2) Chebyshev → type I pass-band ripple
type II stop band ripple

(3) Elliptic → Ripples in both s.b and p.b
Has zeroes.

Design of DT IIR Filters from Analog.

Discretize by using one of many techniques.

$$H_c(s) \longrightarrow H(z)$$

must satisfy:

(1) Imaginary axis → mapped to unit circle

$$s = j\Omega \Rightarrow z = e^{j\omega}$$

(2) stability of $H_c(s)$ should result in stable $H(z)$

Two methods:

(1) Impulse invariance → match impulse

$$h[n] = T h_c(nT)$$

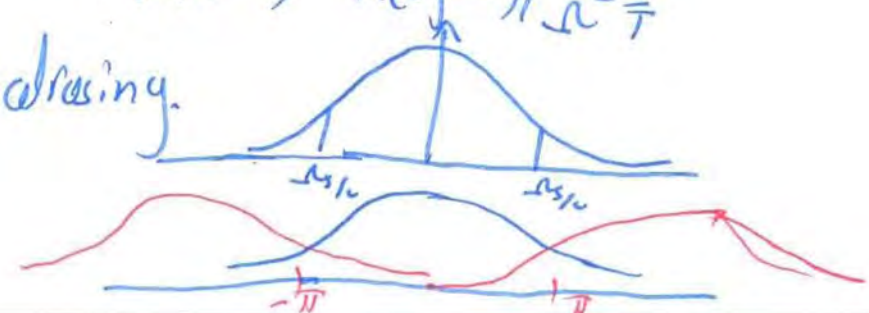
$$\Downarrow$$

$$H(e^{j\omega}) = \sum_{k=-\infty}^{\infty} H_c(j\Omega + j\Omega_s k)\Big|_{\Omega = \frac{\omega}{T}}$$

if $H_c(j\Omega)$ band limited $|\Omega| \leq \Omega_s/2$ then:

$$H(e^{j\omega}) = H_c(j\Omega)\Big|_{\Omega = \frac{\omega}{T}}$$

otherwise → aliasing.



(→) Bilinear transform → basic idea.

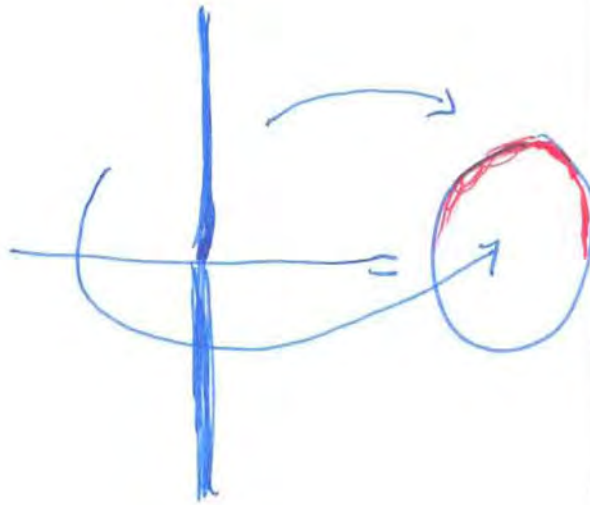
(•) Distorts $H_c(j\Omega)$ such that it is bandlimited.

$$s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}$$

$$z = \frac{1+\frac{T}{2}s}{1-\frac{T}{2}s}$$

for $s=0 \Rightarrow z=1$

$s=\infty \Rightarrow z=-1$



<u>Matlab</u>: butter, cheby1, cheby2, ellip

Next time FIR ...