Generalized linear-phase systems

\[ H(e^{j\omega}) = \alpha e^{j\omega} e^{-j\omega \beta} \]

Real, allow sign change

\[ \text{grad} [H(e^{j\omega})] = \alpha (\text{except when } \alpha e^{j\omega} \text{ changes sign}) \]
Example: 
\((M+1)\)-point moving average

\[
\frac{1}{M+1}
\]

\[
H(e^{j\omega}) = A(e^{j\omega})
\]
Example \((M+1)\)-point moving average

\[ H(e^{j\omega}) = \frac{1}{M+1} \left[ \frac{\sin(\omega (M+1)/2)}{\sin(\omega/2)} \right] e^{-j\omega M/2} \]

\[ A(e^{j\omega}) \]
GLP for FIR $\rightarrow$ must have symmetry

$h[n] = h[\text{ } m-n]$

**Type I** (M even)

**Type II** (M odd)
GUP for FIR → must have symmetry

\[ h[n] = h[m-n] : \]

\underline{Type I} \quad (M \text{ even})

\underline{Type II} \quad (M \text{ odd})

\[ A(e^{j\omega}) \]

1

\[ \pi \]

\[ \frac{1}{2} \]

\[ \pi \]

always
geren

zero
GLP for FIR → must have symmetry

\[ h[n] = h[M-n] : \]

**Type I** \((M \text{ even})\)

\[ A(e^{j\omega}) = h[m] + 2 \sum_{k=1}^{\frac{M}{2}} h^\prime[M-k] \cos(\omega k) \]

**Type II** \((M \text{ odd})\)

\[ A(e^{j\omega}) = \text{In the text} \]
\[ h[n] = -h[M-n] \]

Type III (M even)

\[ A(e^{j\omega}) = j 2 \sum_{k=1}^{M} h[M-k] \sin(\omega k) \]

Type IV (M odd)
Type III \((M \text{ even})\)

\[ h[n] = -h[M-n] \]

\[ A(e^{j\omega}) = j \sum_{k=1}^{M} h\left[\frac{M}{2}-k\right] \sin(\omega k) \]

Type IV \((M \text{ odd})\)

\[ A(e^{j\omega}) = \text{see text} \]

Always \(= 0\)
Zeroes of IIR system

Type I, II: \( h[n] = h[n-M] \)

\[
H(z) = \sum_{n=0}^{M} h[n]z^{-n}
\]
Zeroes of GUP system

Type 1, II: \[ h[n] = h[n-M] \]

\[ H(z) = \sum_{n=0}^{M} h[n]z^{-n} = \]

\[ = \sum_{n=0}^{M} h[M-n]z^{-n} = \]
Zeroes of GI/G system

Type I, II: \( h[n] = h[n-M] \)

\[
H(z) = \sum_{n=0}^{M} h[n] z^{-n} = \sum_{n=0}^{M} h[M-n] z^{-n} = z^{-M} \sum_{n=0}^{M} h[M-n] z^{M-n}
\]
Zeros of CLP system

Type 1, II: \[ h[n] = h[n-M] \]

\[ H(z) = \sum_{n=0}^{M} h[n] z^{-n} = \]

\[ = \sum_{n=0}^{M} h[M-n] z^{-n} = z^{-M} \sum_{n=0}^{M} h[M-n] z^{n-M} \]

\[ = z^{-M} \sum_{k=0}^{M} h[k] z^{k} \]
Zeroes of GLP System

Type I, II: \( h[n] = h[n-M] \)

\[ H(z) = \sum_{n=0}^{M} h[n] z^{-n} = \]

\[ = \sum_{n=0}^{M} h[M-n] z^{-n} = z^{-M} \sum_{n=0}^{M} h[M-n] z^{M-n} \]

\[ = z^{-M} \sum_{k=0}^{M} h[k] z^{k} \]

\[ \Rightarrow H(z) = z^{-M} H(z^{-1}) \]
$$H(z) = z^{-M} H(z^{-1}) \quad \text{Type I, II}$$

$$H(-1) = 0 \quad \text{Type II} \quad \text{(Never high-pass)}$$

For GUP, if $\alpha = re^{j\theta}$ is a zero
\[
\frac{1}{\alpha}^{\text{max}} \text{ is also a zero}
\]
Zeroes of GLP system

Type I, II: \( h[n] = h[n-M] \)

\[
H(z) = \sum_{n=0}^{M} h[n]z^{-n} = \\
= \sum_{n=0}^{M} h[M-n]z^{-n} = z^{-M} \sum_{n=0}^{M} h[M-n]z^{n-M} \\
= z^{-M} \sum_{k=0}^{M} h[k]z^{k} \\
\Rightarrow H(z) = z^{-M} H(z^{M})
\]

For type II; odd

\[
H(-1) = (-1)^{M} H(-1) = -H(1) \Rightarrow H(-1) = 0
\]
Similarly, one can show for type III, IV:

\[ H(z) = -z^{-m} H(z-1) \]

\[ H(1) = 0 \rightarrow \text{Never low-pass} \]

for type III:

\[ H(-1) = 0 \rightarrow \text{Only band-pass} \]
Relation of FIR GLP to min-phase systems

\[ H(z) = H_{min}(z) H_{max}(z) H_{mc}(z) \]

minimum phase

maximum phase
There is much more to learn, ... but...

This is

The END (for now)
What's Next:

EE 121 → digital communication
EE 126 → Prob. & Random processes.
EE 127A → optimization
EE 145B → Image Proc. & tomography

More advanced
EE 125A (EE 123 + EE 126) +
EE 125B
CS 240 Vision
EE 224A, 226A, 227A, 229, CS 181A+B

EE 225E → Principles of MRI
Magnetic Resonance Imaging (MRI) is a non-invasive imaging modality. Unlike Computed Tomography (CT) that uses x-ray, MRI does not use any ionizing radiation and is considered safe. MRI provides a large number of flexible contrast parameters, which give excellent soft-tissue contrast.

The class will cover:

**Fundamentals of MRI:**
- Multi-dimensional Fourier Transforms and linear systems
- Nuclear Magnetic Resonance Physics
- Imaging Sequences
- Contrast Generation
- Image reconstruction
- MRI Hardware and Software
- Imaging tradeoffs and image artifacts

**Advanced Topics:**
- Rapid imaging
- Parallel Imaging
- Emerging research opportunities (High-field, dynamic imaging, functional imaging, hyperpolarization, compressed sensing)

Class includes hands-on Matlab labs for sequence design and reconstruction, tour to an MRI facility and guest lecture by a radiologist.

* This year, for the first time students will perform imaging experiments on a physical earth-field MRI system