(a) From the figure,

\[ y[n] = (x[n] + x[n] * h_1[n]) * h_2[n] \]
\[ = (x[n] * (\delta[n] + h_1[n])) * h_2[n]. \]

Let \( h[n] \) be the impulse response of the overall system,

\[ y[n] = x[n] * h[n]. \]

Comparing with the above expression,

\[ h[n] = (\delta[n] + h_1[n]) * h_2[n] \]
\[ = h_2[n] + h_1[n] * h_2[n] \]
\[ = \alpha^n u[n] + \beta^{(n-1)} u[n - 1]. \]

(b) Taking the Fourier transform of \( h[n] \) from part (a),

\[ H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n} \]
\[ = \sum_{n=-\infty}^{\infty} \alpha^n u[n]e^{-j\omega n} + \beta \sum_{n=-\infty}^{\infty} \alpha^{(n-1)} u[n - 1]e^{-j\omega n} \]
\[ = \sum_{n=0}^{\infty} \alpha^n e^{-j\omega n} + \beta \sum_{\ell=0}^{\infty} \alpha^{(\ell-1)} e^{-j\omega \ell}, \]

where we have used \( \ell = (n - 1) \) in the second sum.

\[ H(e^{j\omega}) = \frac{1}{1 - \alpha e^{-j\omega}} + \frac{\beta e^{-j\omega}}{1 - \alpha e^{-j\omega}} \]
\[ = \frac{1 + \beta e^{-j\omega}}{1 - \alpha e^{-j\omega}}, \text{ for } |\alpha| < 1. \]

Note that the Fourier transform of \( \alpha^n u[n] \) is well known, and the second term of \( h[n] \) (see part (a)) is just a scaled and shifted version of \( \alpha^n u[n] \). So, we could have used the properties of the Fourier transform to reduce the algebra.

(c) We have

\[ H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} \]
\[ = \frac{1 + \beta e^{-j\omega}}{1 - \alpha e^{-j\omega}}, \]

cross multiplying,

\[ Y(e^{j\omega})[1 - \alpha e^{-j\omega}] = X(e^{j\omega})[1 + \beta e^{-j\omega}] \]

taking the inverse Fourier transform, we have

\[ y[n] - \alpha y[n - 1] = x[n] + \beta x[n - 1]. \]

(d) From part (a):

\[ h[n] = 0, \text{ for } n < 0. \]

This implies that the system is CAUSAL.

If the system is stable, its Fourier transform exists. Therefore, the condition for stability is the same as the condition imposed on the frequency response of part (b). That is, STABLE, if \( |\alpha| < 1. \)
\[ y[n] = h[n] \ast (e^{-jw_0 n} x[n]) = \sum_{k=-\infty}^{+\infty} e^{-jw_0 k} x[k] h[n - k]. \]

Let \( x[n] = a x_1[n] + b x_2[n], \) then:

\[
y[n] = h[n] \ast (e^{-jw_0 n} (ax_1[n] + bx_2[n])) \\
= \sum_{k=-\infty}^{+\infty} e^{-jw_0 k} (ax_1[k] + bx_2[k]) h[n - k] \\
= a \sum_{k=-\infty}^{+\infty} e^{-jw_0 k} x_1[k] h[n - k] + b \sum_{k=-\infty}^{+\infty} e^{-jw_0 k} x_2[k] h[n - k] \\
= ay_1[n] + by_2[n]
\]

where \( y_1[n] \) and \( y_2[n] \) are the responses to \( x_1[n] \) and \( x_2[n] \) respectively. We thus conclude that system \( S \) is linear.

(b) Let \( x_2[n] = x[n - n_0], \) then:

\[
y_2[n] = h[n] \ast (e^{-jw_0 n} x_2[n]) \\
= \sum_{k=-\infty}^{+\infty} e^{-jw_0 (n - k)} x_2[n - k] h[k] \\
= \sum_{k=-\infty}^{+\infty} e^{-jw_0 (n - k)} x[n - n_0 - k] h[k] \\
\neq y[n - n_0].
\]

We thus conclude that system \( S \) is not time invariant.

(c) Since the magnitude of \( e^{-jw_0 n} \) is always bounded by 1 and \( h[n] \) is stable, a bounded input \( x[n] \) will always produce a bounded input to the stable LTI system and therefore the output \( y[n] \) will be bounded. We thus conclude that system \( S \) is stable.

(d) We can rewrite \( y[n] \) as:

\[
y[n] = h[n] \ast (e^{-jw_0 n} x[n]) \\
= \sum_{k=-\infty}^{+\infty} e^{-jw_0 (n - k)} x[n - k] h[k] \\
= \sum_{k=-\infty}^{+\infty} e^{-jw_0 n} e^{jw_0 k} x[n - k] h[k] \\
= e^{-jw_0 n} \sum_{k=-\infty}^{+\infty} e^{jw_0 k} x[n - k] h[k].
\]

System \( C \) should therefore be a multiplication by \( e^{-jw_0 n}. \)
\[
\begin{align*}
\begin{bmatrix} 0 & u \\ 1 & 0 \end{bmatrix} = [u]_y
\end{align*}
\]

By writing out the convolution, we get:

\[
[y - u]_y \sum_{k=0}^{\infty} = [u]_y
\]

Since the system is LTI, we have:

\[
\left|\left(\tau_1^x\right)X\right| = \left(\tau_1^x\right)X(\tau_1^x)_*X = \left(\tau_1^x\right)^x
\]

Therefore:

\[
[u - ]_*x = [u]_d
\]

We therefore have:

\[
[u]x * [u - ]_*x = [u]x[u - ]_*x \sum_{k=-\infty}^{\infty} = [u]^x
\]

Using the change of variable \( r = -k \), we can rewrite \( [u]_r \) as:

\[
2.59. (a)
\]
\[
[y - y]_0 0 = [y + y]_0 = [y]x
\]

Therefore,

\[
u_0 0 = u - u
\]

\[
\sum_{z=0}^{\infty} [y - y]_0 = (z)x
\]

\[
[z - z - I]_0 = z - z = (z)x
\]

\[
I < |z|
\]

\[
\frac{z - z - I}{z - z} = (z)x
\]

(c)

Which is stable.

\[
[z + z + z]_0 \sum_{z=0}^{\infty} [y - y]_0 (z) = (z)x
\]

Therefore,

\[
[z - z - I]_0 \sum_{z=0}^{\infty} [y - y]_0 = (z)x
\]

\[
I = |z|
\]

ROC includes \( z \) and

\[
t = \frac{i(z) + yz}{\gamma (z)} \sum_{z=0}^{\infty} = (z) = (z)x
\]

(q)

\[
[z + z - z]_0 \left( \frac{z}{z - z} \right) (z - z) = (z)x
\]

\[
\left( \frac{z - z}{z - z} \right) \sum_{z=0}^{\infty} = \frac{z(z - z)}{z - z} = (z)x
\]

\[
(z)X \left( z - z \right) \Rightarrow [z, z - z]x
\]

\[
(z)X \frac{z - z}{z} \Rightarrow [u]x
\]
\[ \frac{(z + \frac{\varepsilon}{\delta})(z^2 + za - 2)z}{3/8} = \]

\[ \frac{1 - z \left( \frac{\varepsilon}{\delta} + 1 - z - \varepsilon - z \right)}{1 - z \varepsilon - z} = (1 - z)x_{\varepsilon - z} = (z)X \iff \]

\[ [(\varepsilon - u) - x = [\varepsilon + u - x = [u]h] \]

\[ \frac{1}{3} < |z| \]

\[ \frac{1}{3} + z \left( \frac{\varepsilon}{\delta} + z - \varepsilon \right) = (z)X \]

---

From the pole-zero diagram.
\[
\frac{(\frac{\xi}{\tau} + 1)(\frac{\xi}{\tau} - 1)}{(1)(1 + \frac{\xi}{\tau})} + \frac{(1 - \frac{\xi}{\tau})(\frac{\xi}{\tau} - \frac{\xi}{\tau})}{(\frac{\xi}{\tau} - 1)(1 + \frac{\xi}{\tau})} + \frac{(1 - \frac{\xi}{\tau})(\frac{\xi}{\tau} + \frac{\xi}{\tau})}{(\frac{\xi}{\tau})(1 + \frac{\xi}{\tau})} = \]

\[
\frac{(1 - z)(\frac{\xi}{\tau} + z)(\frac{\xi}{\tau} - z)}{z \rho u z (1 + z) (I + z)} \oint \frac{2\pi i}{I} = [I] y
\]

Therefore, using a contour $C$ that lies outside of $I$, we get

\[
1 < |z| \quad \frac{1 - z - I}{2} \cdot \frac{(1 - z \frac{\xi}{\tau} + I)(1 - z \frac{\xi}{\tau} - I)}{z - z + 1 - z} = (z) x
\]
The system is stable because the ROC includes the unit circle. It is also causal since $h[n]$ for $n = 0$ is 0.

\[ [1 - u] x - [u] x = [1 - u] \frac{\hat{p}}{\epsilon} - [u] \hat{y} \]

\[ [1 - u] n \begin{pmatrix} \frac{\hat{p}}{\epsilon} \\ \hat{z} \end{pmatrix} \hat{z} - [u] n \begin{pmatrix} \frac{\hat{p}}{\epsilon} \\ \hat{z} \end{pmatrix} = [u] \hat{y} \]

\[ \frac{\hat{p}}{\epsilon} < |z| \quad \frac{1 - \frac{\hat{p} z^2}{\epsilon^2} - \frac{\hat{z}}{\epsilon}}{1 - \frac{\hat{z}^2}{\epsilon^2} - \frac{\hat{p}}{\epsilon}} = \frac{(z) X}{(z) A} = (z) H \]

\[ \frac{\hat{p}}{\epsilon} < |z| \quad \frac{1 - \frac{\hat{p} z^2}{\epsilon^2} - \frac{\hat{z}}{\epsilon}}{9} - \frac{1 - \frac{\hat{z}^2}{\epsilon^2} - \frac{\hat{p}}{\epsilon}}{9} = (z) A \]

\[ \frac{\hat{z}}{\epsilon} > |z| > \frac{\hat{z}}{1} \quad \frac{1 - \frac{\hat{z}^2}{\epsilon^2} - \frac{\hat{p}}{\epsilon}}{1} - \frac{1 - \frac{\hat{z}}{\epsilon} - \frac{\hat{p}}{1}}{1} = (z) X \]
\[
\left\{ \left( \frac{\mu - 2}{\mu} \right) \frac{1}{\lambda} \right\} \frac{2}{1} H + \left( \frac{z}{m \lambda} \right) \frac{1}{\lambda} H \right\} \frac{2}{1} = (m \lambda)^2 H
\]

\[
(m \lambda)^2 X \left( m \lambda \right)^2 H =
\]

\[
(m \lambda)^2 X \left\{ \left( \frac{\mu - 2}{\mu} \right) \frac{1}{\lambda} H + \left( \frac{z}{m \lambda} \right) \frac{1}{\lambda} H \right\} \frac{2}{1} =
\]

\[
\left\{ \left( \frac{\mu - 2}{\mu} \right) \frac{1}{\lambda} H \left( \frac{\mu - 2}{\mu} \right) \frac{1}{\lambda} H \right\} \frac{2}{1} = (m \lambda)^2 H
\]

\[
\left\{ \left( \frac{z}{\mu \lambda} \right) \frac{1}{\lambda} H \left( \frac{z}{\mu \lambda} \right) \frac{1}{\lambda} H \right\} \frac{2}{1} = (m \lambda)^2 H
\]

\[
(m \lambda)^2 H (m \lambda)^2 X = (m \lambda)^2 X
\]

\[
\text{downsampled by } 2:
\]

4.29. We can analyze the system in the frequency domain.
\[ n \geq |m| > \frac{\xi}{\mu} \quad \frac{\xi}{\mu} > |m| \geq \frac{\xi}{\mu} \quad \frac{\xi}{\mu} \geq |m| \geq 0 \quad 0 \geq |m| \] \}

\( = (\beta_\varphi)^2 H \)

\[ \frac{3}{5} = \frac{\varphi}{5} \Rightarrow \frac{300}{5} \cdot 1000 = 2m \Rightarrow L = 2m \]

\[ \frac{3}{5} = \frac{\varphi}{5} \Rightarrow \frac{300}{5} \cdot 500 = 2m \Rightarrow L = 2m \]

Now that \( T \) is set, figure out \( H \) band edges.

\( T = 1/3000 \) s.

Since only half the frequency band of \( \mathcal{X}(\mathcal{U}) \) is needed, we can alias everything past \( U = 2000\pi \). Hence,

\[ |\mathcal{U}| \leq 2000\pi \quad 1000\pi \leq |\mathcal{U}| \leq 2000\pi \quad |\mathcal{U}| \leq 4000\pi \quad |\mathcal{U}| = 0 = (\mathcal{X}(\mathcal{U})^2) X \]
\[
\left( \frac{z}{y} - u \frac{z}{\nu \lambda} \right) \cos = \\
\left( \left( \frac{z}{y} - u \frac{z}{\nu \lambda} \right) \cos + \left( \frac{z}{y} - u \frac{z}{\nu \lambda} \right) \sin \right) \frac{z}{y} = \\
u \left( \frac{z}{y} \right) \cos \left( \frac{z}{y} \right) H \left( \frac{y}{x} \right) \cos \frac{z}{y} = \left[ \cos \right]
\]

Therefore, since \( \left( m \right) H \) is an LTI system, we can analyze the responses to each of the two eigenfunctions separately.

\[
u \left( \frac{z}{y} \right) \cos \left( \frac{z}{y} \right) H \left( \frac{y}{x} \right) \cos \frac{z}{y} = \\
\left[ \left( \frac{z}{y} - u \frac{z}{\nu \lambda} \right) \cos + \left( \frac{z}{y} - u \frac{z}{\nu \lambda} \right) \sin \right] \frac{z}{y} = \left( \frac{z}{y} - u \frac{z}{\nu \lambda} \right) \cos
\]

Note that the choice of \( \lambda \) is not unique:

\[
\lambda = \lambda
\]

Recall the relationship \( U \lambda = m \). We can simplify this in our system conversion:

\[
u \left( \frac{z}{y} \right) \cos \left( \frac{z}{y} \right) H \left( \frac{y}{x} \right) \cos \frac{z}{y} = \left( \nu \lambda \right) H
\]

\[
u \left( \frac{z}{y} \right) \cos \left( \frac{z}{y} \right) H \left( \frac{y}{x} \right) \cos \frac{z}{y} = \left( m \lambda \right) H
\]
This is done with the following system:

Gain = n/3

Therefore, going s[n] to s[n-1] basically requires changing the sampling rate by a factor of 3kHz/5kHz = 3/5.

In both systems, the speech was altered first so that the subsequent sampling results in no aliasing.