8.30. We have a finite length sequence, whose 64-pt DFT contains only one nonzero point (for $k = 32$).

(a) Using the synthesis equation Eq. (8.68):

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]W_N^{-kn}, \quad 0 \le n \le (N-1)$$

Substitution yields:

$$\begin{aligned} x[n] &= \frac{1}{64}X[32]W_{64}^{-32n} \\ &= \frac{1}{64}e^{j\frac{2\pi}{64}(32)n} \\ &= \frac{1}{64}e^{j\pi n} \\ x[n] &= \frac{1}{64}(-1)^n, \quad 0 \le n \le (N-1) \end{aligned}$$

The answer is unique because we have taken the 64-pt DFT of a 64-pt sequence.

(b) The sequence length is now $N = 192$.

$$x[n] = \frac{1}{192}\sum_{k=0}^{191} X[k]W_{192}^{-kn}, \; 0 \le n \le 191$$

$$x[n] = \begin{cases} \frac{1}{64}(-1)^n & 0 \le n \le 63 \\ 0 & 64 \le n \le 191 \end{cases}$$

This solution is not unique. By taking only 64 spectral samples, $x[n]$ will be aliased in time. As an alternate sequence, consider

$$x'[n] = \frac{1}{64}\left(\frac{1}{3}\right)(-1)^n, \quad 0 \le n \le 191$$

8.31. We have a 10-point sequence, $x[n]$. We want a modified sequence, $x_1[n]$, such that the 10-pt. DFT of $x_1[n]$ corresponds to

$$X_1[k] = X(z)\big|_{z=\frac{1}{2}e^{j[(2\pi k/10)+(\pi/10)]}}$$

Recall the definition of the Z-transform of $x[n]$:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

Since $x[n]$ is of finite duration ($N = 10$), we assume:

$$x[n] = \begin{cases} \text{nonzero}, & 0 \le n \le 9 \\ 0, & \text{otherwise} \end{cases}$$

Therefore,

$$X(z) = \sum_{n=0}^{9} x[n]z^{-n}$$

Substituting in $z = \frac{1}{2}e^{j[(2\pi k/10)+(\pi/10)]}$:

$$X(z)\big|_{z=\frac{1}{2}e^{j[(2\pi k/10)+(\pi/10)]}} = \sum_{n=0}^{9} x[n]\left(\frac{1}{2}e^{j[(2\pi k/10)+(\pi/10)]}\right)^{-n}$$

We seek the signal $x_1[n]$, whose 10-pt. DFT is equivalent to the above expression. Recall the analysis equation for the DFT:

$$X_1[k] = \sum_{n=0}^{9} x_1[n]W_{10}^{kn}, \quad 0 \le k \le 9$$

Since $W_{10}^{kn} = e^{-j(2\pi/10)kn}$, by comparison

$$x_1[n] = x[n]\left(\frac{1}{2}e^{j(\pi/10)}\right)^{-n}$$

8.32. We have a finite-length sequence, $x[n]$ with $N = 8$. Suppose we interpolate by a factor of two. That is, we wish to double the size of $x[n]$ by inserting zeros at all odd values of n for $0 \le n \le 15$.

Mathematically,

$$y[n] = \begin{cases} x[n/2], & n \text{ even}, \quad 0 \le n \le 15 \\ 0, & n \text{ odd}, \end{cases}$$

The 16-pt. DFT of $y[n]$:

$$\begin{aligned} Y[k] &= \sum_{n=0}^{15} y[n] W_{16}^{kn}, \quad 0 \le k \le 15 \\ &= \sum_{n=0}^{7} x[n] W_{16}^{2kn} \end{aligned}$$

Recall, $W_{16}^{2kn} = e^{j(2\pi/16)(2k)n} = e^{-j(2\pi/8)kn} = W_8^{kn}$,

$$Y[k] = \sum_{n=0}^{7} x[n] W_8^{kn}, \quad 0 \le k \le 15$$

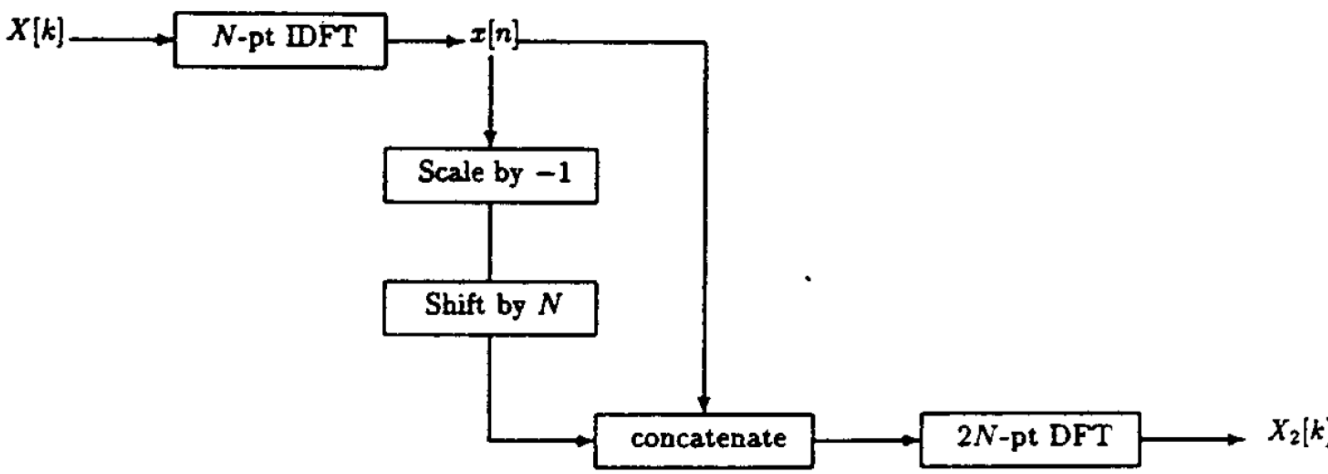
Therefore, the 16-pt. DFT of the interpolated signal contains two copies of the 8-pt. DFT of $x[n]$. This is expected since $Y[k]$ is now periodic with period 8 (see problem 8.1). Therefore, the correct choice is C.

As a quick check, $Y[0] = X[0]$.

8.33. (a) Since

$$x_2[n] = \begin{cases} x[n], & 0 \le n \le N-1 \\ -x[n-N], & N \le n \le 2N-1 \\ 0, & \text{otherwise} \end{cases}$$

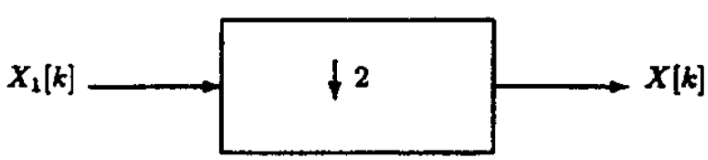
If $X[k]$ is known, $x_2[n]$ can be constructed by :



(b) To obtain $X[k]$ from $X_1[k]$, we might try to take the inverse DFT (2N-pt) of $X_1[k]$, then take the N-pt DFT of $x_1[n]$ to get $X[k]$.
However, the above approach is highly inefficient. A more reasonable approach may be achieved if we examine the DFT analysis equations involved. First,

$$\begin{aligned} X_1[k] &= \sum_{n=0}^{2N-1} x_1[n] W_{2N}^{kn}, \qquad 0 \le k \le (2N-1) \\ &= \sum_{n=0}^{N-1} x[n] W_{2N}^{kn} \\ &= \sum_{n=0}^{N-1} x[n] W_{N}^{(k/2)n}, \quad 0 \le k \le (N-1) \\ X_1[k] &= X[k/2], \quad 0 \le k \le (N-1) \end{aligned}$$

Thus, an easier way to obtain $X[k]$ from $X_1[k]$ is simply to decimate $X_1[k]$ by two.



8.34. (a) The DFT of the even part of a real sequence:

If $x[n]$ is of length N, then $x_e[n]$ is of length $2N-1$:

$$
\begin{aligned}
x_e[n] &= \begin{cases} \frac{x[n]+x[-n]}{2}, & (-N+1) \le n \le (N-1) \\ 0 & \text{otherwise} \end{cases} \\
X_e[k] &= \sum_{n=-N+1}^{N-1} \left(\frac{x[n]+x[-n]}{2}\right) W_{2N-1}^{kn}, \quad (-N+1) \le k \le (N-1) \\
&= \sum_{n=-N+1}^{0} \frac{x[-n]}{2} W_{2N-1}^{kn} + \sum_{n=0}^{N-1} \frac{x[n]}{2} W_{2N-1}^{kn}
\end{aligned}
$$

Let $m = -n$,

$$
\begin{aligned}
X_e[k] &= \sum_{n=0}^{N-1} \frac{x[n]}{2} W_{2N-1}^{-kn} + \sum_{n=0}^{N-1} \frac{x[n]}{2} W_{2N-1}^{kn} \\
X_e[k] &= \sum_{n=0}^{N-1} x[n] \cos\left(\frac{2\pi kn}{2N-1}\right)
\end{aligned}
$$

Recall

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{kn}, \quad 0 \le k \le (N-1)$$

and

$$Re\{X[k]\} = \sum_{n=0}^{N-1} x[n] \cos\left(\frac{2\pi kn}{N}\right)$$

So: $\text{DFT}\{x_e[n]\} \ne Re\{X[k]\}$

(b)

$$
\begin{aligned}
Re\{X[k]\} &= \frac{X[k]+X^*[k]}{2} \\
&= \frac{1}{2}\sum_{n=0}^{N-1} x[n] W_N^{kn} + \frac{1}{2}\sum_{n=0}^{N-1} x[n] W_N^{-kn} \\
&= \frac{1}{2}\sum_{n=0}^{N-1} (x[n] + x[N-n]) W_N^{kn}
\end{aligned}
$$

So,

$$Re\{X[k]\} = \text{DFT}\left\{\frac{1}{2}(x[n] + x[N-n])\right\}$$

8.35. From condition 1, we can determine that the sequence is of finite length ($N = 5$). Given:

$$\begin{aligned} X(e^{j\omega}) &= 1 + A_1 \cos\omega + A_2 \cos 2\omega \\ &= 1 + \frac{A_1}{2}(e^{j\omega} + e^{-j\omega}) + \frac{A_2}{2}(e^{j2\omega} + e^{-j2\omega}) \end{aligned}$$

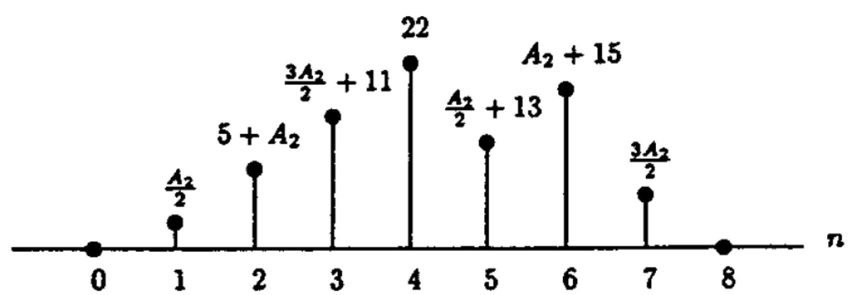
From the Fourier analysis equation, we can see by matching terms that:

$$x[n] = \delta[n] + \frac{A_1}{2}(\delta[n-1] + \delta[n+1]) + \frac{A_2}{2}(\delta[n-2] + \delta[n+2])$$

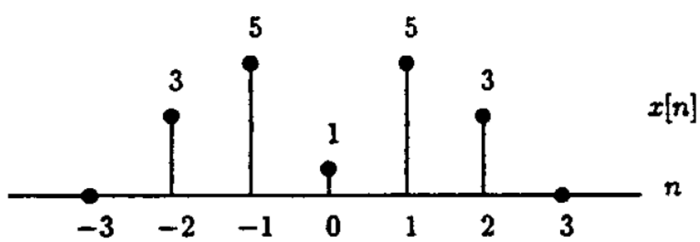
Condition 2 yields one of the values for the amplitude constants of condition 1. Since $x[n] * \delta[n-3] = x[n-3] = 5$ for $n = 2$, we know $x[-1] = 5$, and also that $x[1] = x[-1] = 5$. Knowing both these values tells us that $A_1 = 10$.

For condition 3, we perform a circular convolution between $x[((n-3))_8]$ and $w[n]$, a three-point sequence. For this case, linear convolution is the same as circular convolution since $N = 8 \geq 6 + 3 - 1$.

We know $x[((n-3))_8] = x[n-3]$, and convolving this with $w[n]$ from Fig P8.35-1 gives:



For $n = 2, w[n] * x[n-3] = 11$ so $A_2 = 6$. Thus, $x[2] = x[-2] = 3$, and we have fully specified $x[n]$:



8.36. We have the finite-length sequence:

$$x[n] = 2\delta[n] + \delta[n-1] + \delta[n-3]$$

(i) Suppose we perform the 5-pt DFT:

$$X[k] = 2 + W_5^k + W_5^{3k}, \quad 0 \le k \le 5$$

where $W_5^k = e^{-j(\frac{2\pi}{5})k}$.

(ii) Now, we square the DFT of $x[n]$:

$$\begin{aligned} Y[k] &= X^2[k] \\ &= 2 + 2W_5^k + 2W_5^{3k} \\ &\quad + 2W_5^k + W_5^{2k} + W_5^{5k} \\ &\quad + 2W_5^{3k} + W_5^{4k} + W_5^{6k}, \quad 0 \le k \le 5 \end{aligned}$$

Using the fact $W_5^{5k} = W_5^0 = 1$ and $W_5^{6k} = W_5^k$

$$Y[k] = 3 + 5W_5^k + W_5^{2k} + 4W_5^{3k} + W_5^{4k}, \quad 0 \le k \le 5$$

(a) By inspection,

$$y[n] = 3\delta[n] + 5\delta[n-1] + \delta[n-2] + 4\delta[n-3] + \delta[n-4], \quad 0 \le n \le 5$$

(b) This procedure performs the autocorrelation of a real sequence. Using the properties of the DFT, an alternative method may be achieved with convolution:

$$y[n] = \text{IDFT}\{X^2[k]\} = x[n] * x[n]$$

The IDFT and DFT suggest that the convolution is circular. Hence, to ensure there is no aliasing, the size of the DFT must be $N \ge 2M - 1$ where M is the length of $x[n]$. Since $M = 3, N \ge 5$.

8.43. (a) Overlap add:

If we divide the input into sections of length L, each section will have an output length:

$$L + 100 - 1 = L + 99$$

Thus, the required length is

$$L = 256 - 99 = 157$$

If we had 63 sections, $63 \times 157 = 9891$, there will be a remainder of 109 points. Hence, we must pad the remaining data to 256 and use another DFT.

Therefore, we require 64 DFTs and 64 IDFTs. Since $h[n]$ also requires a DFT, the total:

$$\text{65 DFTs and 64 IDFTs}$$

(b) Overlap save:

We require 99 zeros to be padded in from of the sequence. The first 99 points of the output of each section will be discarded. Thus the length after padding is 10099 points. The length of each section overlap is $256 - 99 = 157 = L$.

We require $65 \times 157 = 10205$ to get all 10099 points. Because $h[n]$ also requires a DFT:

$$\text{66 DFTs and 65 IDFTs}$$

(c) Ignoring the transients at the beginning and end of the direct convolution, each output point requires 100 multiplies and 99 adds.

overlap add:

$$\begin{aligned} \#\,\text{mult} &= 129(1024) = 132096 \\ \#\,\text{add} &= 129(2048) = 264192 \end{aligned}$$

overlap save:

$$\begin{aligned} \#\,\text{mult} &= 131(1024) = 134144 \\ \#\,\text{add} &= 131(2048) = 268288 \end{aligned}$$

direct convolution:

$$\begin{aligned} \#\,\text{mult} &= 100(10000) = 1000000 \\ \#\,\text{add} &= 99(10000) = 990000 \end{aligned}$$