7.32. (a) By using Parseval's theorem,

$$\begin{aligned} \epsilon^2 &= \frac{1}{2\pi}\int_{-\pi}^{\pi} |E(e^{j\omega})|^2 d\omega \\ &= \sum_{n=-\infty}^{\infty} |e[n]|^2 \end{aligned}$$

where

$$e[n] = \begin{cases} h_d[n], & n < 0, \\ h_d[n] - h[n], & 0 \le n \le M, \\ h_d[n], & n > M \end{cases}$$

(b) Since we only have control over $e[n]$ for $0 \le m \le M$, we get that ϵ^2 is minimized if $h[n] = h_d[n]$ for $0 \le n \le M$.

(c)

$$w[n] = \begin{cases} 1, & 0 \le n \le M, \\ 0, & \text{otherwise.} \end{cases}$$

which is a rectangular window.

7.34. (a) It is well known that convolving two rectangular windows results in a triangular window. Specifically, to get the $(M+1)$ point Bartlett window for M even, we can convolve the following rectangular windows.

$$\begin{aligned} r_1[n] &= \begin{cases} \sqrt{\frac{2}{M}}, & n = 0, \ldots, \frac{M}{2} - 1 \\ 0, & \text{otherwise} \end{cases} \\ r_2[n] &= r_1[n-1] \end{aligned}$$

Using the known transform of a rectangular window we have

$$\begin{aligned} W_{R_1}(e^{j\omega}) &= \sqrt{\frac{2}{M}} \frac{\sin(\omega M/4)}{\sin(\omega/2)} e^{-j\omega\left(\frac{M}{4} - \frac{1}{2}\right)} \\ W_{R_2}(e^{j\omega}) &= \sqrt{\frac{2}{M}} \frac{\sin(\omega M/4)}{\sin(\omega/2)} e^{-j\omega\left(\frac{M}{4} + \frac{1}{2}\right)} \\ W_B(e^{j\omega}) &= W_{R_1}(e^{j\omega}) W_{R_2}(e^{j\omega}) \\ &= \frac{2}{M} \left(\frac{\sin(\omega M/4)}{\sin(\omega/2)} \right)^2 e^{-j\omega M/2} \end{aligned}$$

Note: The Bartlett window as defined in the text is zero at $n = 0$ and $n = M$. These points are included in the $M + 1$ points.
For M odd, the Bartlett window is the convolution of

$$\begin{aligned} r_3[n] &= \begin{cases} \sqrt{\frac{2}{M}}, & n = 0, \ldots, \frac{M-1}{2} \\ 0, & \text{otherwise} \end{cases} \\ r_4[n] &= \begin{cases} \sqrt{\frac{2}{M}}, & n = 1, \ldots, \frac{M-1}{2} \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

In the frequency domain we have

$$\begin{aligned} W_{R_3}(e^{j\omega}) &= \sqrt{\frac{2}{M}} \frac{\sin(\omega(M+1)/4)}{\sin(\omega/2)} e^{-j\omega\left(\frac{M-1}{4}\right)} \\ W_{R_4}(e^{j\omega}) &= \sqrt{\frac{2}{M}} \frac{\sin(\omega(M-1)/4)}{\sin(\omega/2)} e^{-j\omega\left(\frac{M-3}{4} + 1\right)} \\ W_B(e^{j\omega}) &= W_{R_3}(e^{j\omega}) W_{R_4}(e^{j\omega}) \\ &= \frac{2}{M} \left(\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} \right) \left(\frac{\sin[\omega(M-1)/2]}{\sin(\omega/2)} \right) e^{-j\omega M/2} \end{aligned}$$

(b)

$$w[n] = \left[A + B\cos\left(\frac{2\pi n}{M}\right) + C\cos\left(\frac{4\pi n}{M}\right)\right] w_R[n]$$

$$\begin{aligned} W(e^{j\omega}) &= \left\{2\pi A\delta(\omega) + B\pi\left[\delta\left(\omega + \frac{2\pi}{M}\right) + \delta\left(\omega - \frac{2\pi}{M}\right)\right] + C\pi\left[\delta\left(\omega + \frac{4\pi}{M}\right) + \delta\left(\omega - \frac{4\pi}{M}\right)\right]\right\} \\ &\quad \frac{\otimes}{2\pi}\left\{\frac{\sin(\omega(M+1)/2))}{\sin(\omega/2)} e^{-j\omega M/2}\right\} \end{aligned}$$

where $\otimes$ denotes periodic convolution.

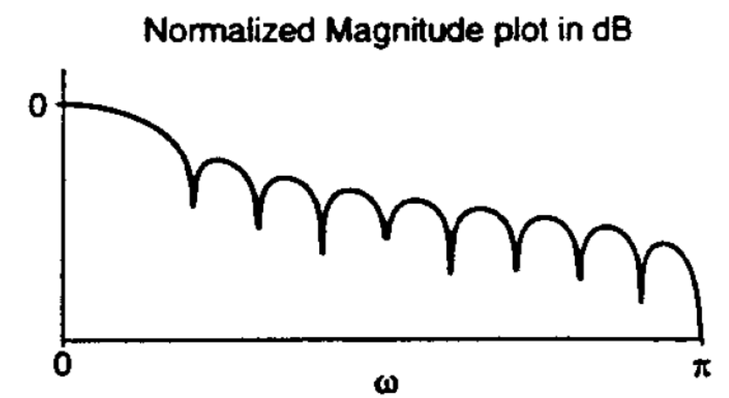
(c) For the Hanning window $A = 0.5$, $B = -0.5$, and $C = 0$.

$$\begin{aligned} w_{\text{Hanning}}[n] &= \left[0.5 - 0.5\cos\left(\frac{2\pi n}{M}\right)\right] w_r[n] \\ W_{\text{Hanning}}(e^{j\omega}) &= 0.5W_R(e^{j\omega}) - 0.25W_R(e^{j\omega}) \otimes \left[\delta\left(\omega + \frac{2\pi}{M}\right) + \delta\left(\omega - \frac{2\pi}{M}\right)\right] \\ &= 0.5W_R(e^{j\omega}) - 0.25\left[W_R(e^{j(\omega + \frac{2\pi}{M})}) + W_R(e^{j(\omega - \frac{2\pi}{M})})\right] \end{aligned}$$

where

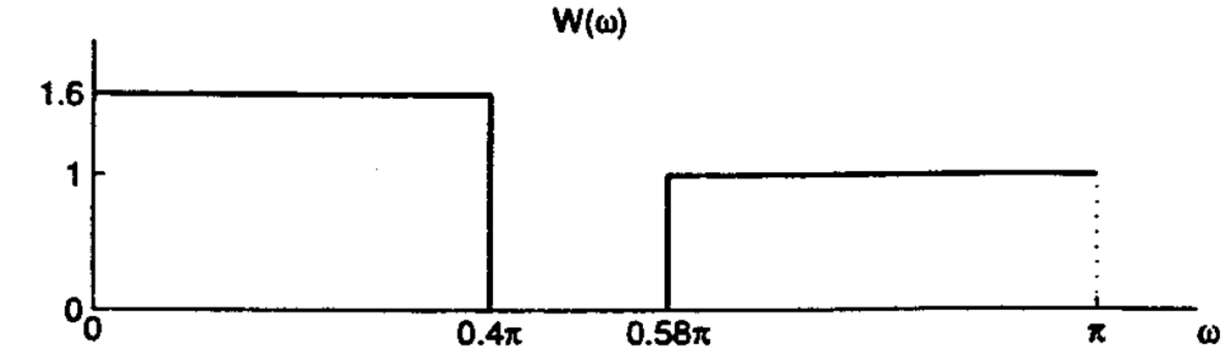
$$W_R(e^{j\omega}) = \frac{\sin(\omega(M+1)/2))}{\sin(\omega/2)} e^{-j\omega M/2}$$

Below is a normalized sketch of the magnitude response in dB.

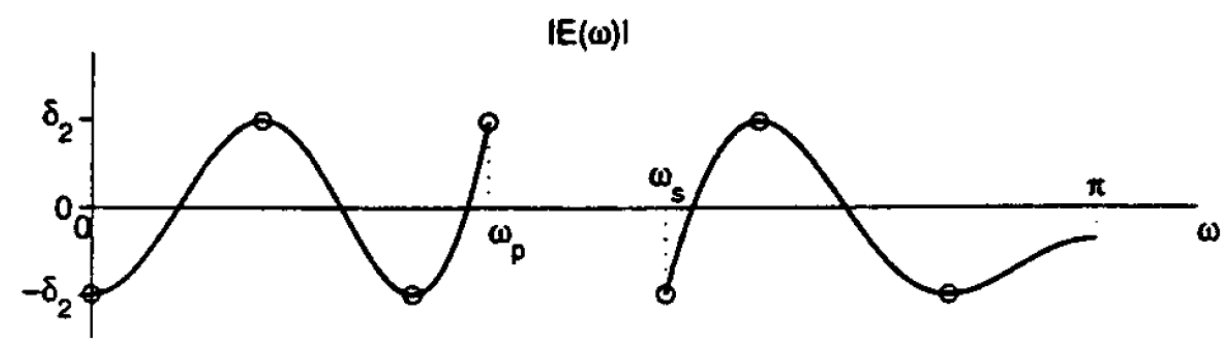


7.36. (a) Since $H(e^{j0}) \neq 0$ and $H(e^{j\pi}) \neq 0$, this must be a Type I filter.

(b) With the weighting in the stopband equal to 1, the weighting in the passband is $\frac{\delta_2}{\delta_1}$.



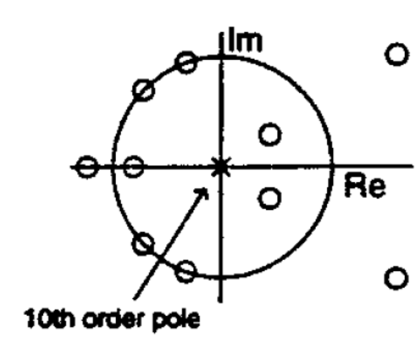
(c)



(d) An optimal (in the Parks-McClellan sense) Type I lowpass filter can have either $L+2$ or $L+3$ alternations. The second case is true only when an alternation occurs at all band edges. Since this filter does not have an alternation at $\omega = \pi$ it should only have $L+2$ alternations. From the figure, we see that there are 7 alternations so $L = 5$. Thus, the filter length is $2L + 1 = 11$ samples long.

(e) Since the filter is 11 samples long, it has a delay of 5 samples.

(f) Note the zeroes off the unit circle are implied by the dips in the frequency response at the indicated frequencies.



7.38. (a) A Type-I lowpass filter that is optimal in the Parks-McClellan can have either $L+2$ or $L+3$ alternations. The second case is true only when an alternation occurs at all band edges. Since this filter does not have an alternation at $\omega = 0$ it only has $L+2$ alternations. From the figure we see there are 9 alternations so $L = 7$. Thus, $M = 2L = 2(7) = 14$.

(b) We have

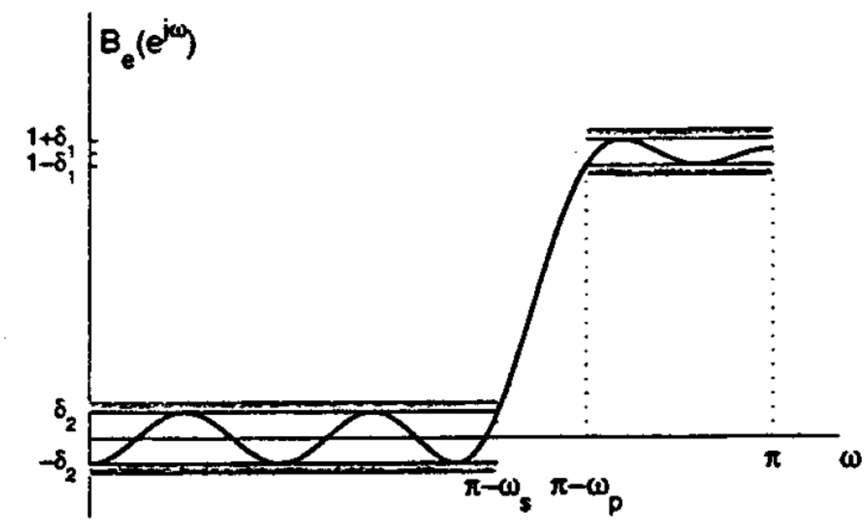
$$\begin{aligned} h_{HP}[n] &= -e^{j\pi n} h_{LP}[n] \\ H_{HP}(e^{j\omega}) &= -H_{LP}(e^{j(\omega-\pi)}) \\ &= -A_e(e^{j(\omega-\pi)})e^{-j(\omega-\pi)\frac{M}{2}} \\ &= A_e(e^{j(\omega-\pi)})e^{-j\omega\frac{M}{2}} \\ &= B_e(e^{j\omega})e^{-j\omega\frac{M}{2}} \end{aligned}$$

where

$$B_e(e^{j\omega}) = A_e(e^{j(\omega-\pi)})$$

The fact that $M = 14$ is used to simplify the exponential term in the third line above.

(c)



(d) *The assertion is correct.* The original amplitude function was optimal in the Parks-McClellan sense. The method used to create the new filter did not change the filter length, transition width, or relative ripple sizes. All it did was slide the frequency response along the frequency axis creating a new error function $E'(\omega) = E(\omega - \pi)$. Since translation does not change the Chebyshev error ($\max|E(\omega)|$) the new filter is still optimal.

7.39. For this filter, $N = 3$, so the polynomial order L is

$$L = \frac{N-1}{2} = 1$$

Note that $h[n]$ must be a type-I FIR generalized linear phase filter, since it consists of three samples, and $H(e^{j\omega}) \neq 0$ for $\omega = 0$. $h[n]$ can therefore be written in the form

$$h[n] = a\delta[n] + b\delta[n-1] + a\delta[n-2]$$

Taking the DTFT of both sides gives

$$\begin{aligned} H(e^{j\omega}) &= a + be^{-j\omega} + ae^{-j2\omega} \\ &= e^{-j\omega}(ae^{j\omega} + b + ae^{-j\omega}) \\ &= e^{-j\omega}(b + 2a\cos w) \\ A(e^{j\omega}) &= b + 2a\cos w \end{aligned}$$

The filter must have at least $L+2=3$ alternations, but no more than $L+3=4$ alternations to satisfy the alternation theorem, and therefore be optimal in the minimax sense. Four alternations can be obtained if all four band edges are alternation frequencies such that the frequency response overshoots at $\omega = 0$, undershoots at $\omega = \frac{\pi}{3}$, overshoots at $\omega = \frac{\pi}{2}$, and undershoots at $\omega = \pi$.

Let the error in the passband and the stopband be δ_p and δ_s. Then,

$$\begin{aligned} A(e^{j\omega})\big|_{\omega=0} &= 1 + \delta_p \\ A(e^{j\omega})\big|_{\omega=\pi/3} &= 1 - \delta_p \\ A(e^{j\omega})\big|_{\omega=\pi/2} &= \delta_s \\ A(e^{j\omega})\big|_{\omega=\pi} &= -\delta_s \end{aligned}$$

Using $A(e^{j\omega}) = b + 2a\cos w$,

$$\begin{aligned} A(e^{j\omega})\big|_{\omega=0} &= b + 2a \\ A(e^{j\omega})\big|_{\omega=\pi/3} &= b + a \\ A(e^{j\omega})\big|_{\omega=\pi/2} &= b \\ A(e^{j\omega})\big|_{\omega=\pi} &= b - 2a \end{aligned}$$

Solving these systems of equations for a and b gives

$$\begin{aligned} a &= \frac{2}{5} \\ b &= \frac{2}{5} \end{aligned}$$

Thus, the optimal (in the minimax sense) causal 3-point lowpass filter with the desired passband and stopband edge frequencies is

$$h[n] = \frac{2}{5}\delta[n] + \frac{2}{5}\delta[n-1] + \frac{2}{5}\delta[n-2]$$